Contaminant Source Identification in River Networks

A Probabilistic Analysis and Application to Altamaha River

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MESL

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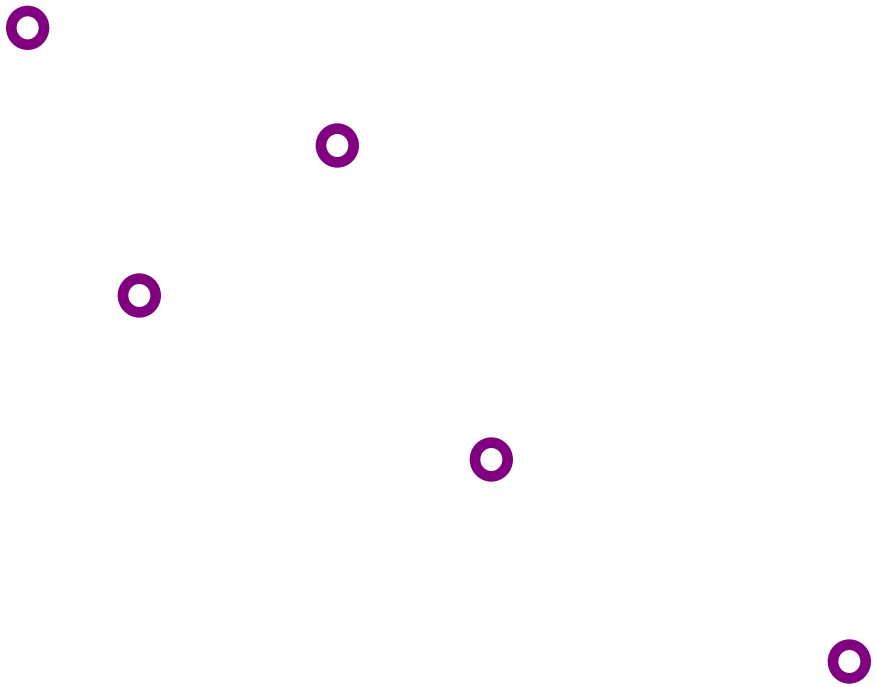
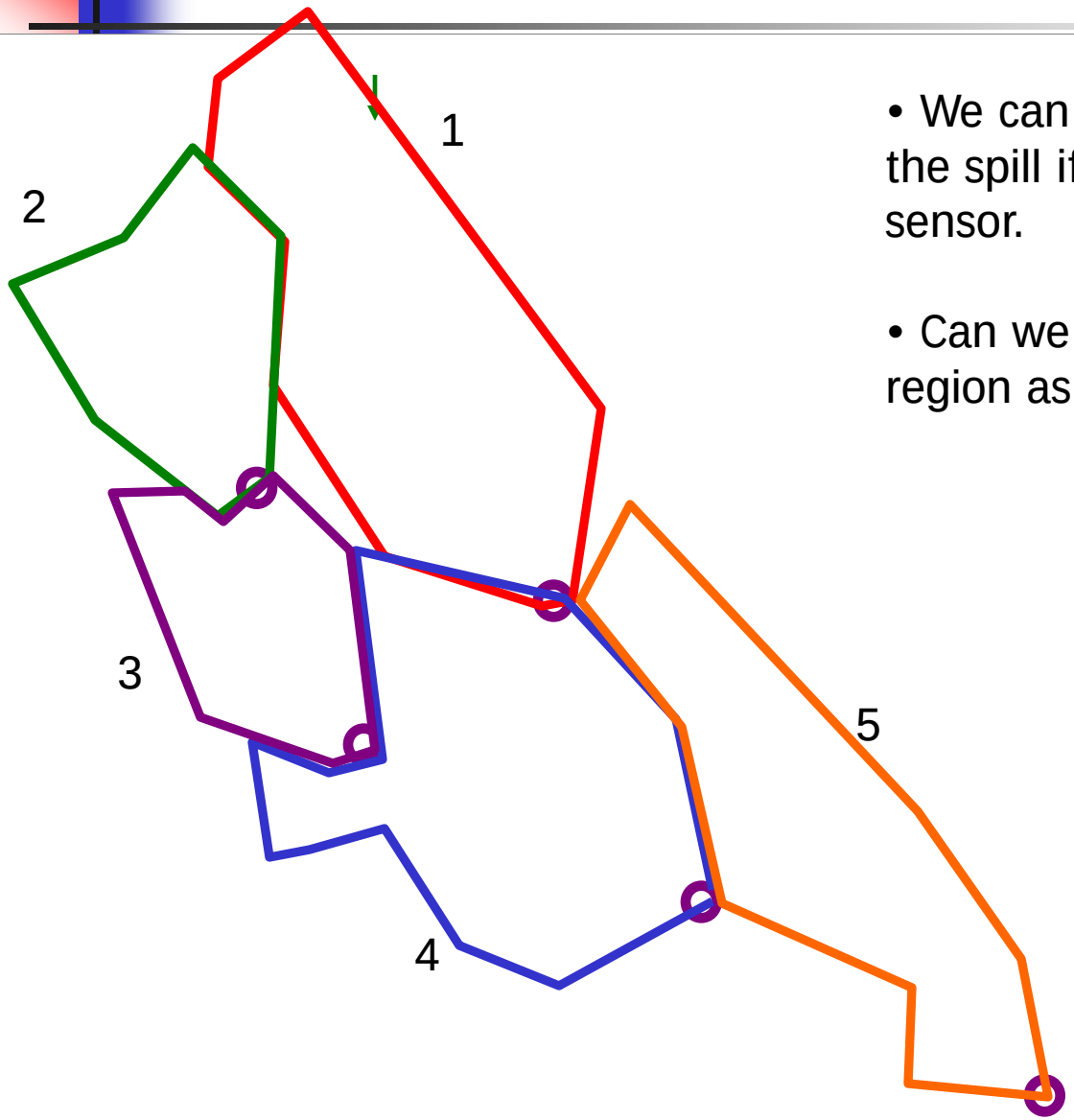
Problem

Given the monitoring network, can we determine the location of the spill?



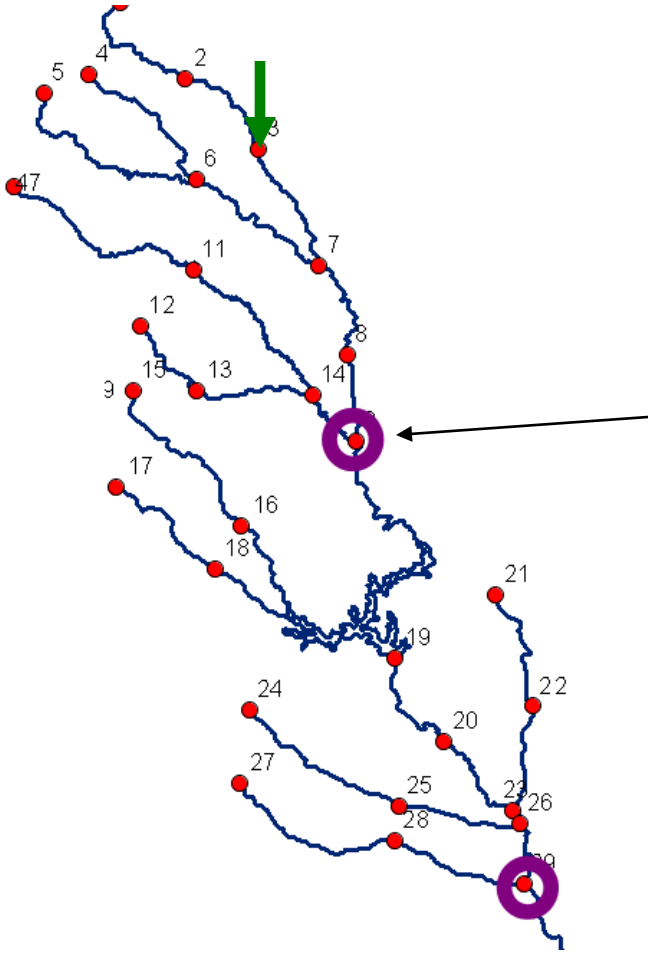
Problem

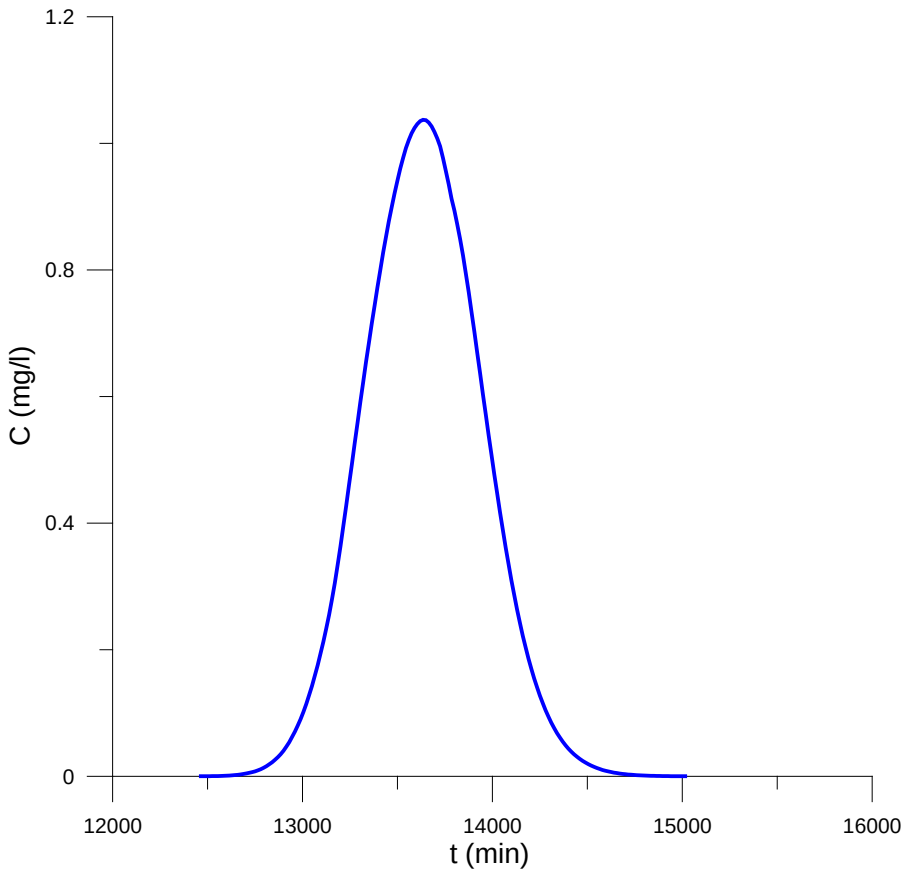


- We can easily determine a region for the spill if we know the first activated sensor.

- Can we find a set of junctions in that region as possible spill locations?

Spill Example

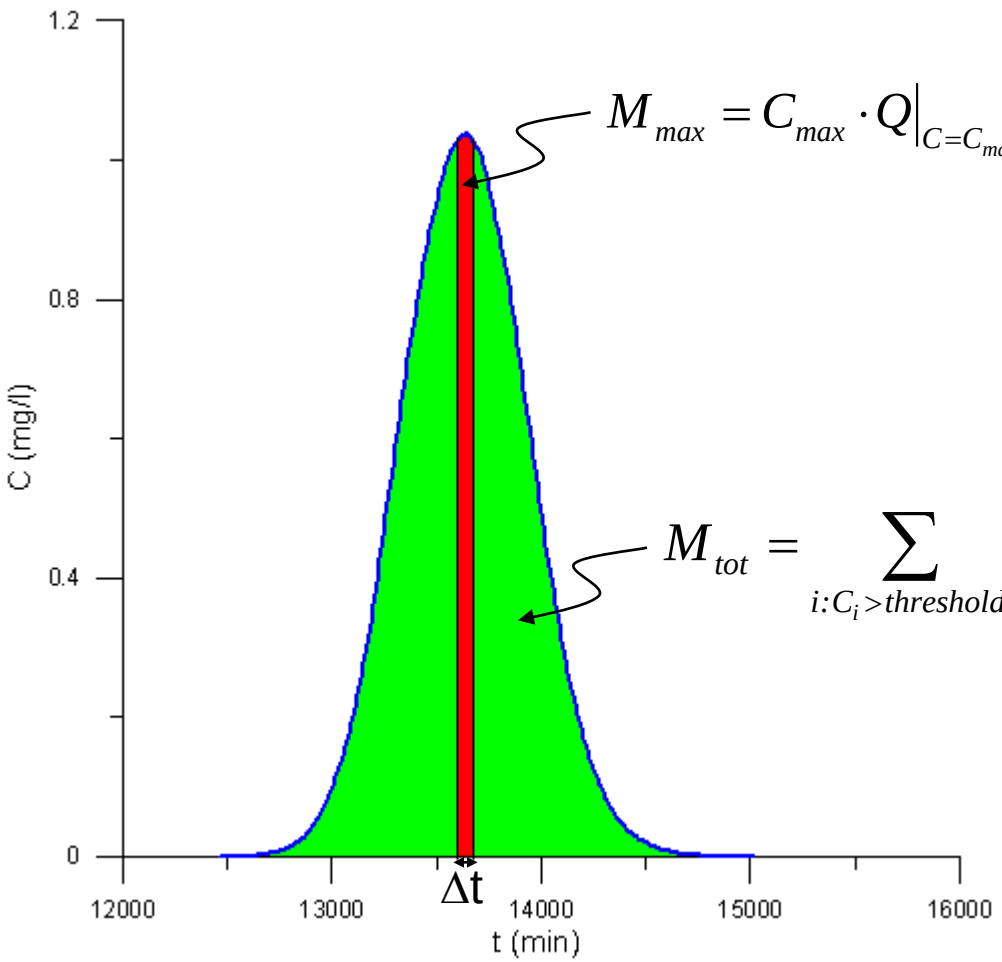




Concentration Profile measured at J9

Parameterization of Concentration Profile

Parameter 1: M_{max}/M_{tot}



$$\frac{M_{max}}{M_{tot}} = \frac{C_{max} \cdot Q\big|_{C=C_{max}}}{\sum_{i:C_i > threshold} C_i \cdot Q_i}$$

Assuming steady flow:

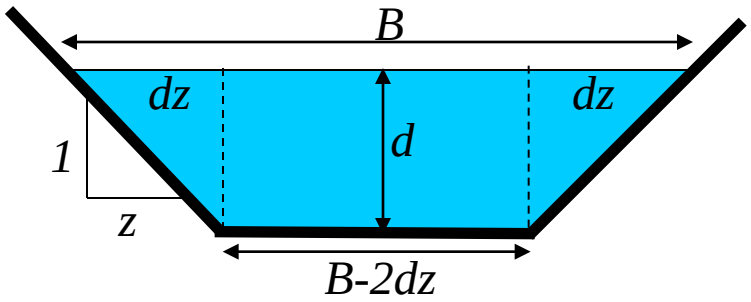
$$\frac{M_{max}}{M_{tot}} = \frac{C_{max}}{\sum_{i:C_i > threshold} C_i}$$

Therefore, this parameter gives information about the vertical scale of the concentration profile.

Parameterization of Concentration Profile

Parameter 2: Peclet Number

$$Pe = \frac{\left(\sum_{k:C_k > threshold} \frac{1}{v_k} v_k \Delta t \right)^2}{\sum_{k:C_k > threshold} \frac{D_k}{v_k^3} v_k \Delta t}$$



$$D = 0.011 \frac{v^2 B^2}{dU_*} \text{ (Fisher et al. 1979)}$$

$$\frac{B^2}{dU_*} = \frac{B^2}{d\sqrt{gRS_o}}$$

$$Pe = \frac{\left(\sum_{k:C_k > threshold} \Delta t \right)^2}{\sum_{k:C_k > threshold} 0.011 \left(\frac{B^2}{dU_*} \right)_k \Delta t}$$

$$\frac{B^2}{dU_*} = \frac{B^2}{d\sqrt{g\left(\frac{Bd - d^2 z}{2\sqrt{d^2 z^2 + d^2} + B - 2dz} \right) S_o}}$$

$$\frac{B^2}{dU_*} \propto \frac{L^2}{L^{3/2}} \text{ assuming } g \text{ and } S_o \text{ as constants.}$$

v : velocity

D : Dispersion coefficient

B : Top width

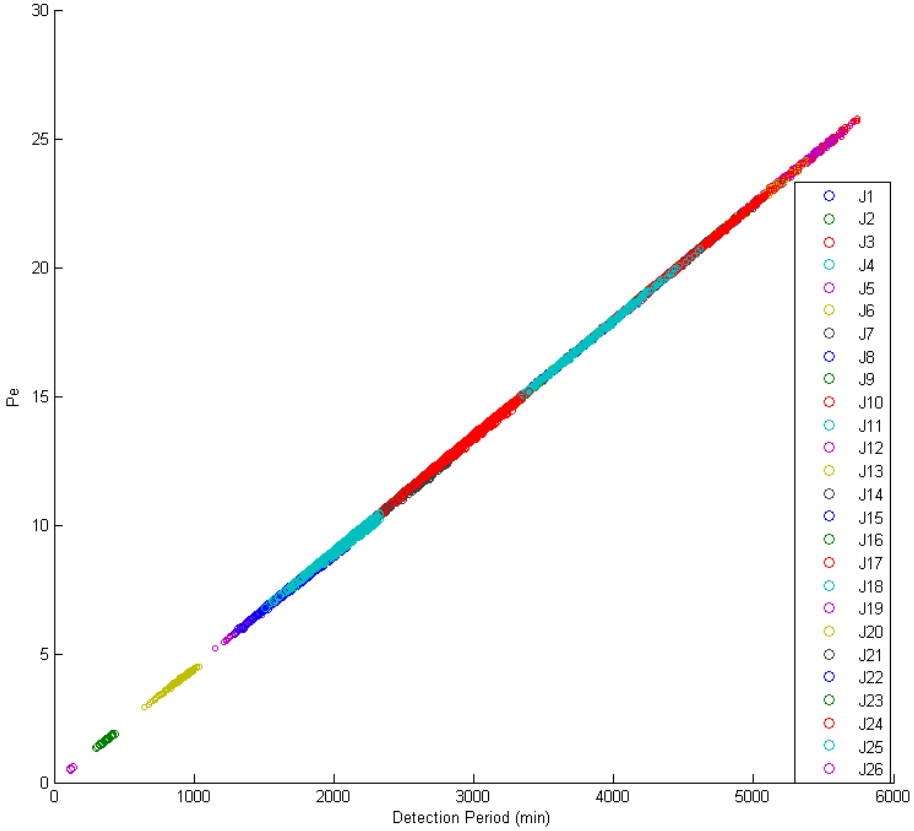
d : depth of flow

U_* : Shear velocity

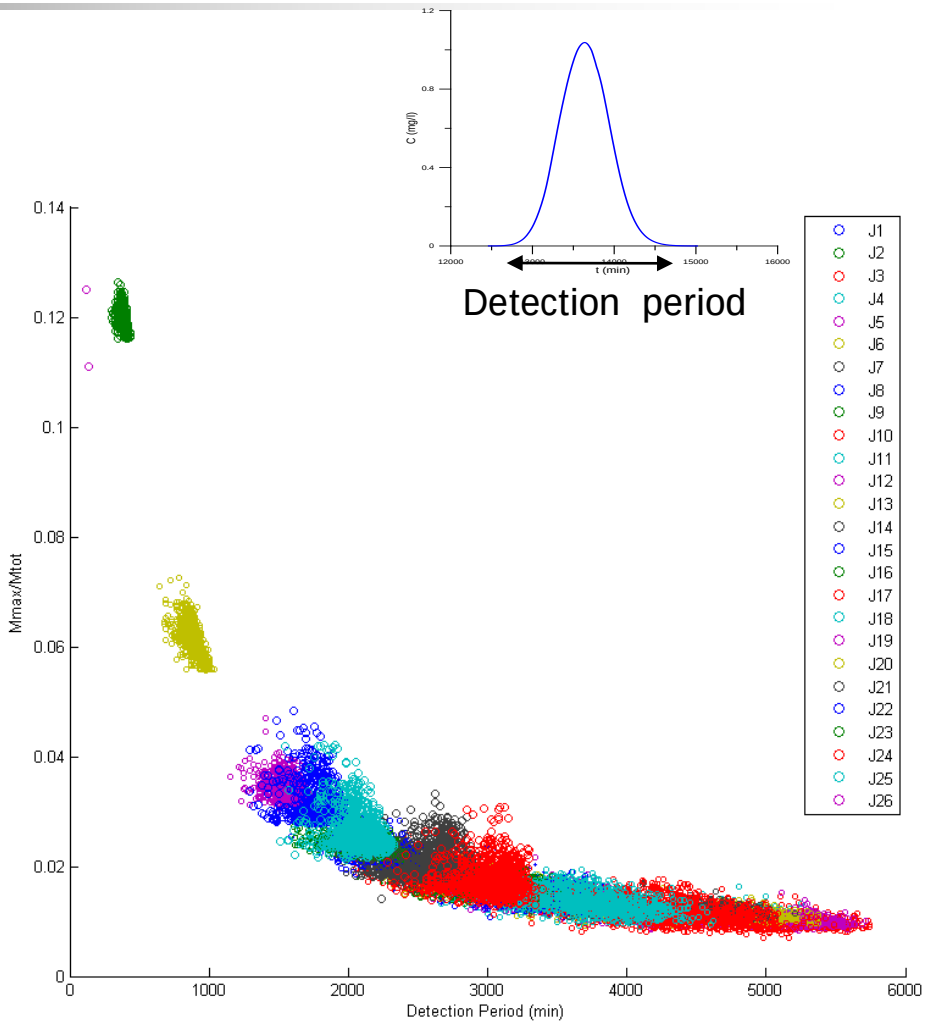
Therefore for small changes in depth,

$$Pe \propto \sum_{k:C_k > threshold} \Delta t = \text{Detection period}$$

Comparison of the two parameters

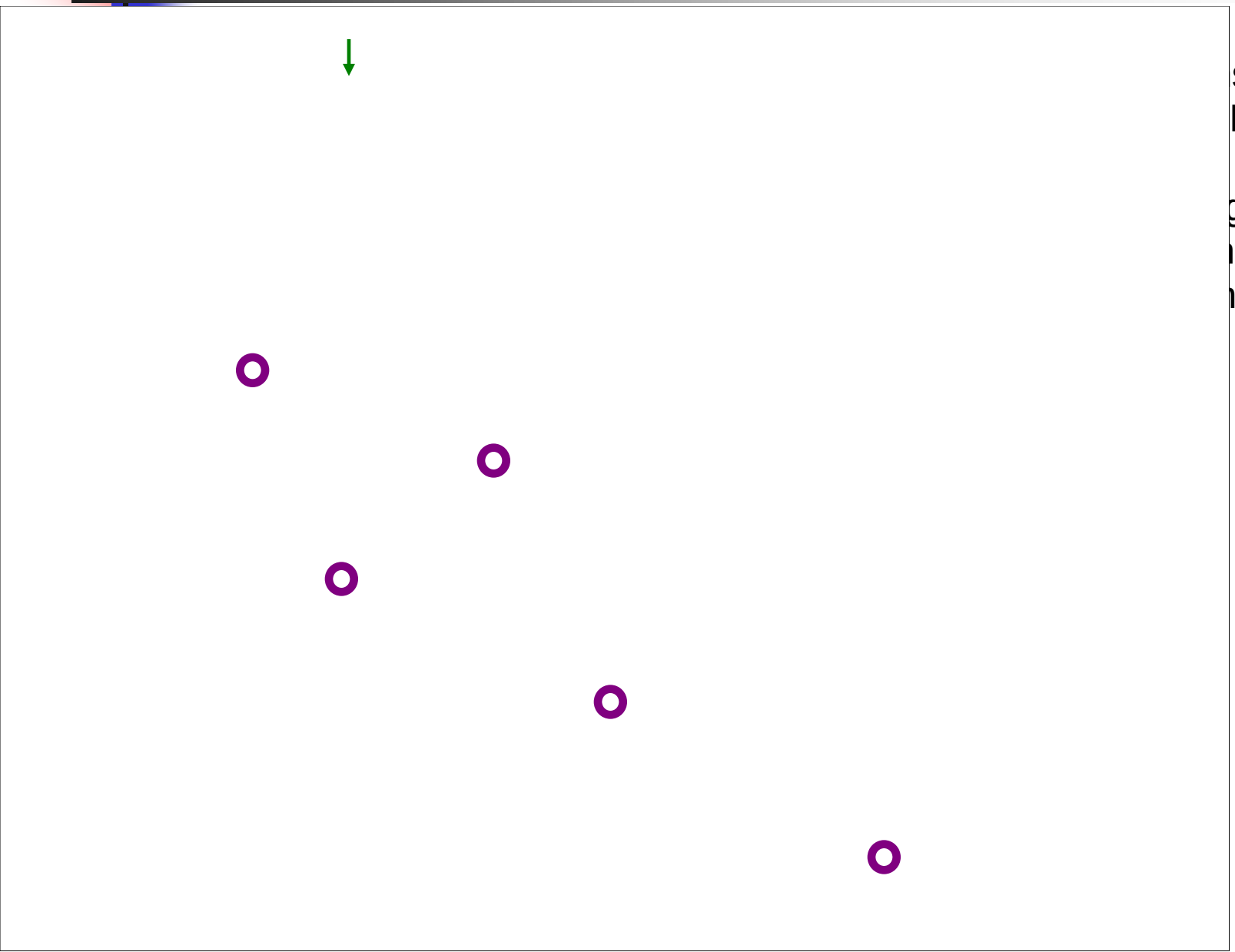


Pe gives information about support base (or detection period)



Mmax/Mtot gives information about vertical scale of the profile

Scenario Generation



s
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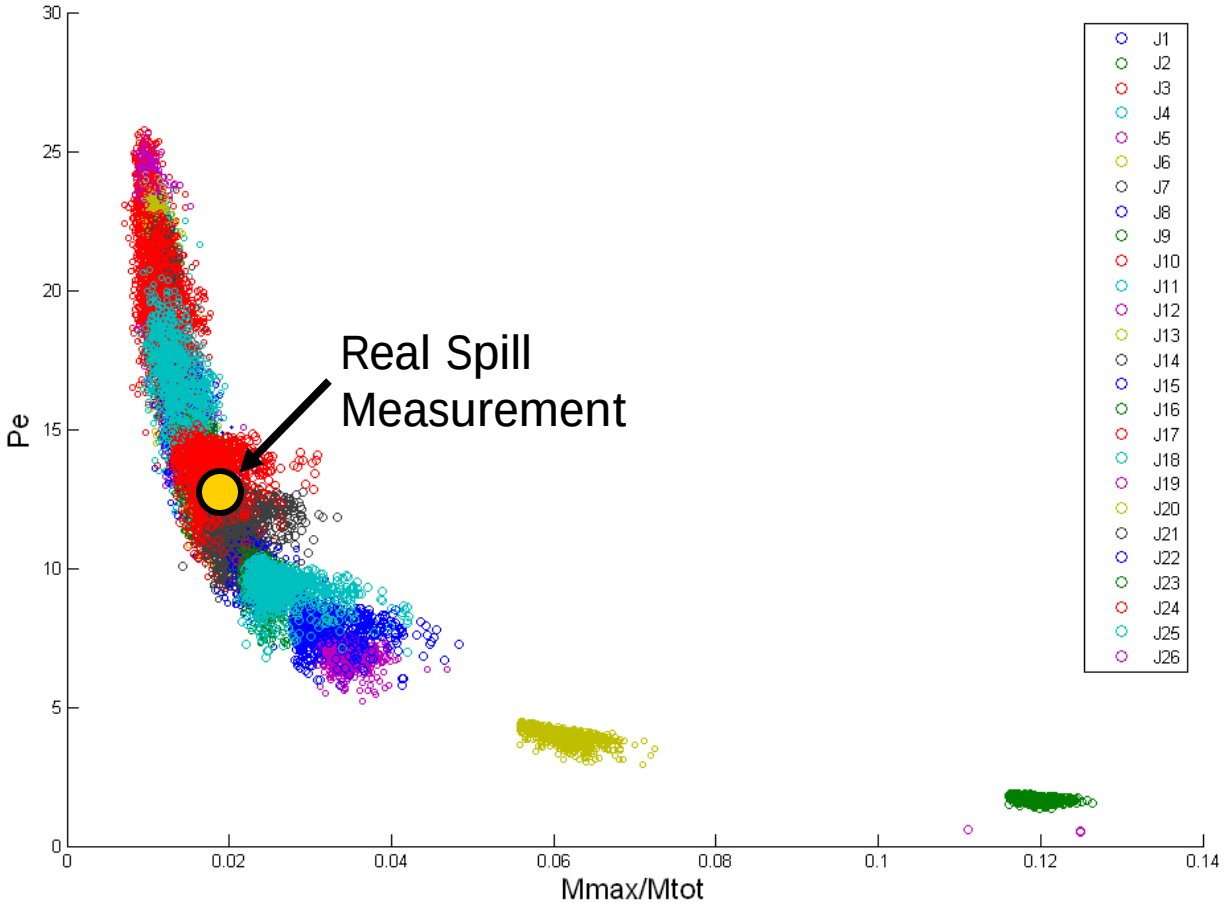
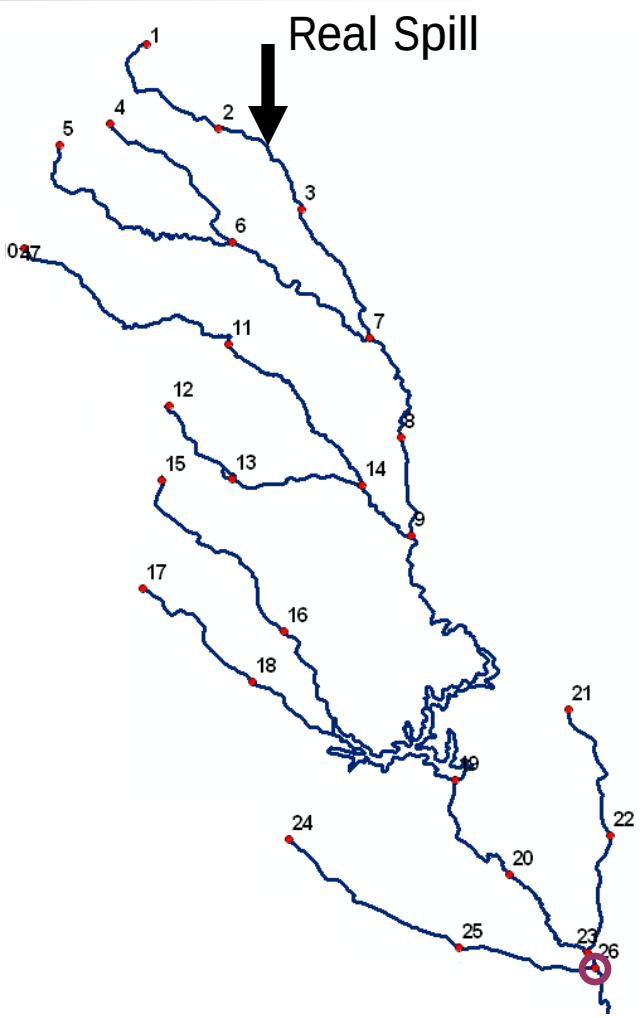
10 days.

Characterization of Scenario Results in terms of Parameter 1 and 2

Region 1 scenarios observed at sensor 1





Random Variables

In our analysis a spill must have three main properties:

X_1: M_{max}/M_{tot} observed at the monitoring station.

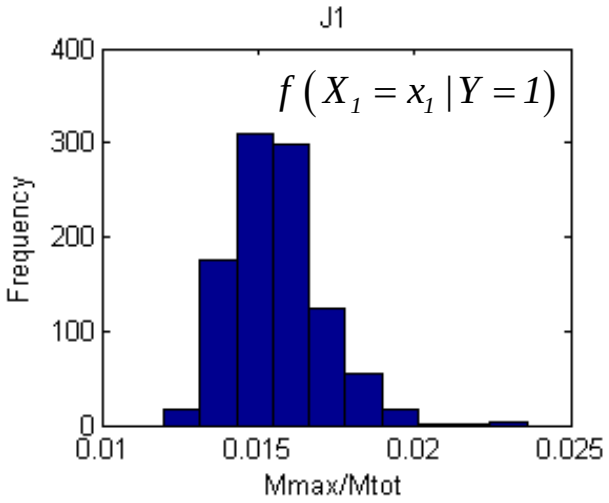
X_2: Pe observed at the monitoring station.

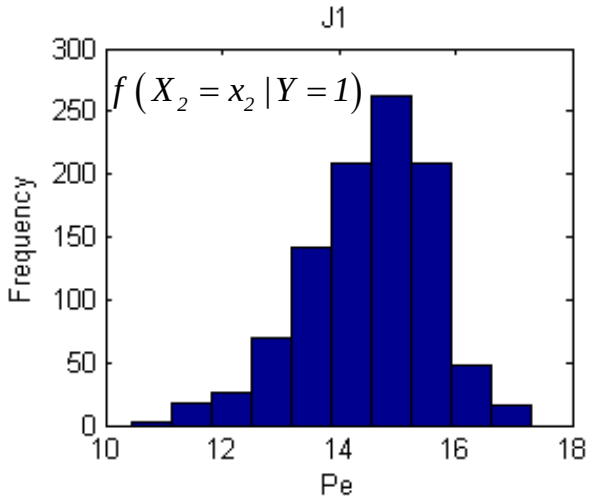
} Features of the spill event

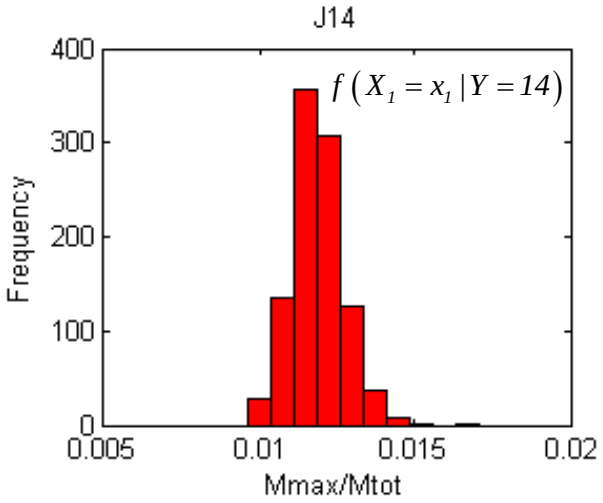
Y: Junction where the spill has occured.

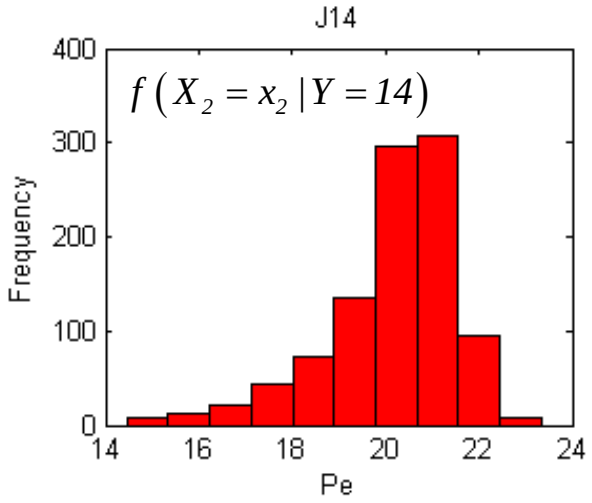
} Class of the spill event

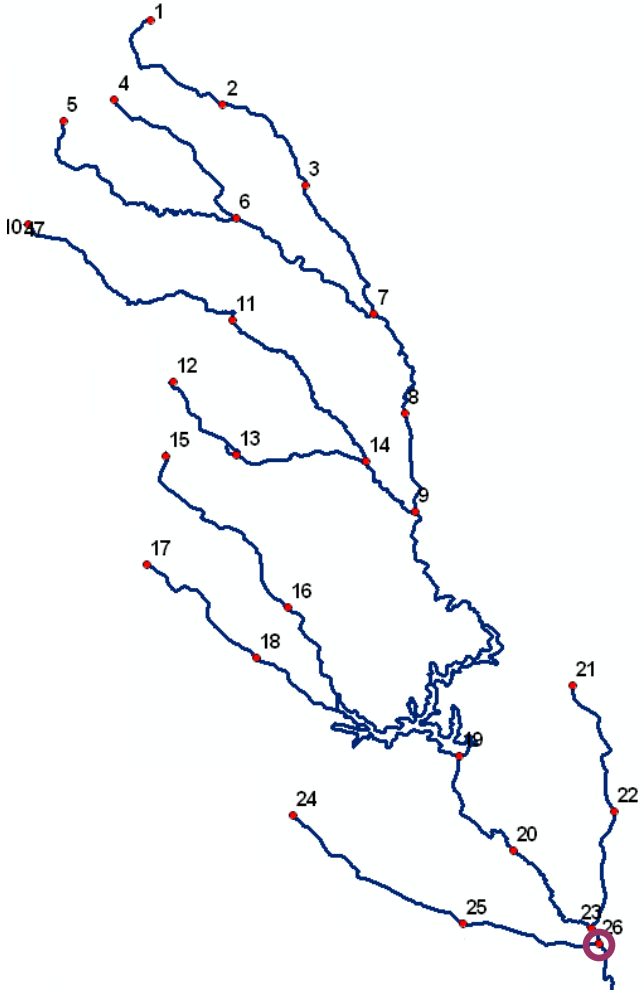
Conditional prior probability densities









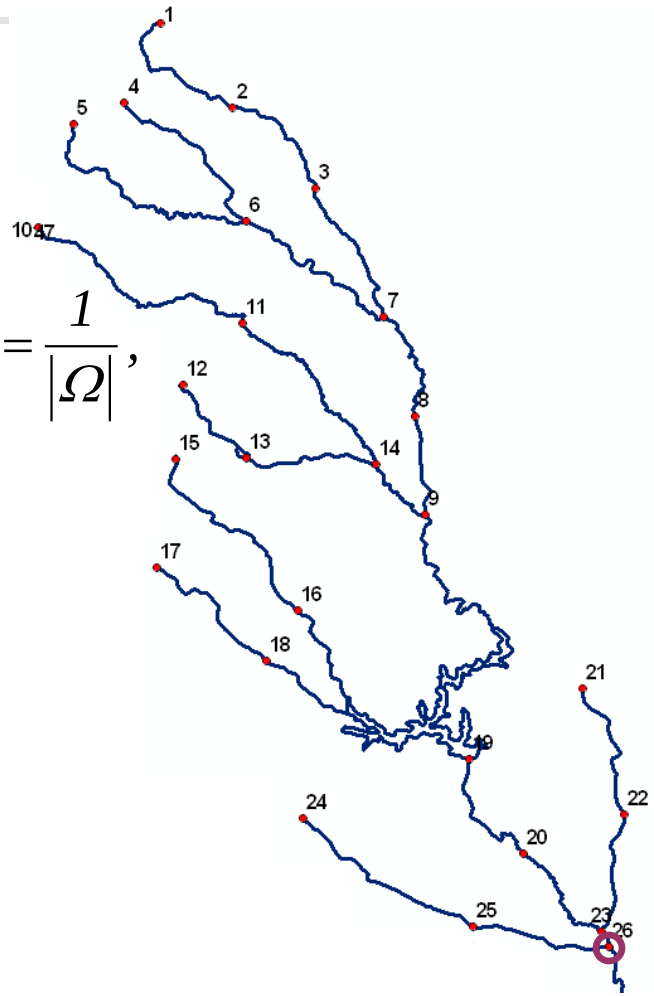


$f\left(X \mid Y\right)$: Conditional probability density function of $X \in \left\{X_1, X_2\right\}$ given that spill occured at Y approximated from Kernel density estimation.

Conditional posterior probabilities

Bayes' Theorem:

Let $\Omega = \{1,2,...,M\}$ is the set of all classes (junctions) possible for a spill event. Then, for $j \in \Omega$ and $P(Y = j) = \frac{1}{|\Omega|}$,



$$P(Y = j \mid X_1 = x_1) = \frac{f(X_1 = x_1 \mid Y = j)}{\sum_{i \in \Omega} f(X_1 = x_1 \mid Y = i)}$$

$$P(Y = j \mid X_2 = x_2) = \frac{f(X_2 = x_2 \mid Y = j)}{\sum_{i \in \Omega} f(X_2 = x_2 \mid Y = i)}$$

$P(Y \mid X)$: Conditional probability of $Y \in \Omega$ given that spill has feature of $X \in \{X_1, X_2\}$

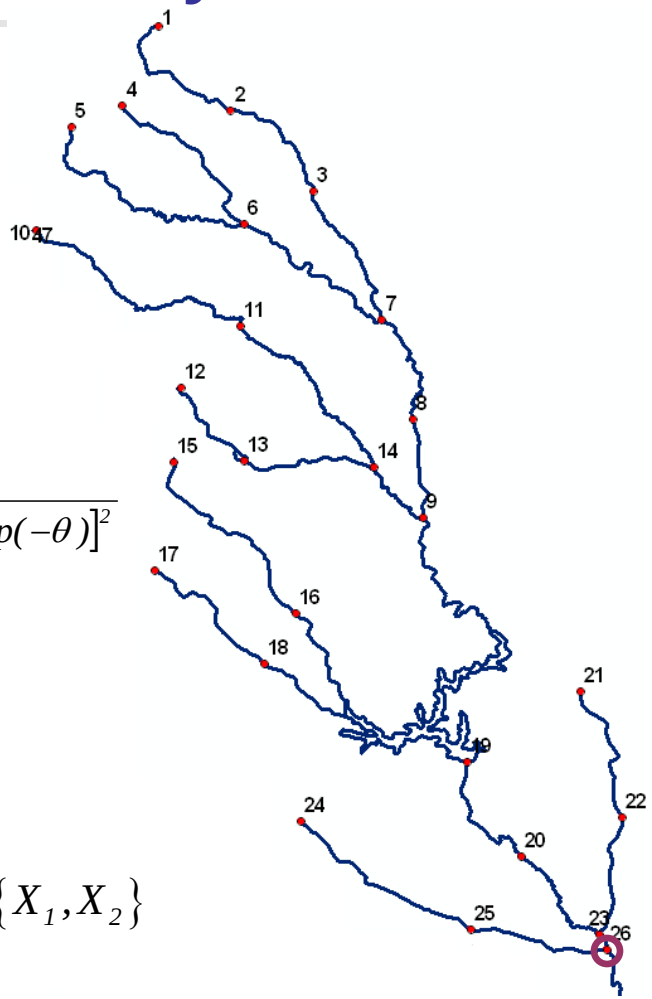
Conditional prior joint probability densities

Copula Concept:

$$f(X_1, X_2 \mid Y) = c(F(X_1 \mid Y), F(X_2 \mid Y)) f(X_1 \mid Y) f(X_2 \mid Y)$$

Frank Copula:

$$c(F_X(x), F_Y(y)) = -\frac{\theta \exp(-\theta(F_X(x) + F_Y(y)))(\exp(-\theta) - 1)}{\left[\exp(-\theta(F_X(x) + F_Y(y)) - \exp(-\theta F_X(x) - \exp(-\theta F_Y(y)) + \exp(-\theta)\right]^2}$$



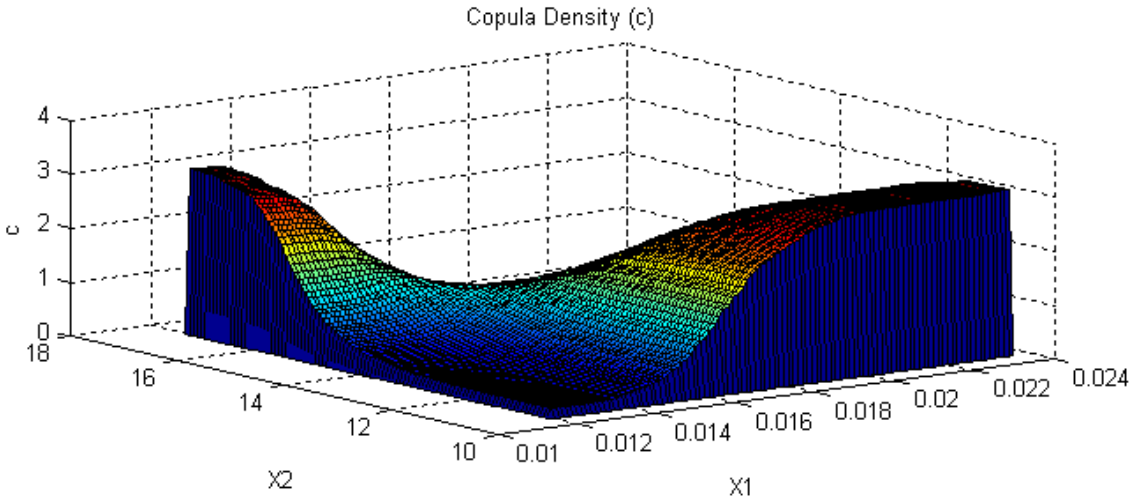
$c(u,v)$: Copula density function, θ : copula parameter

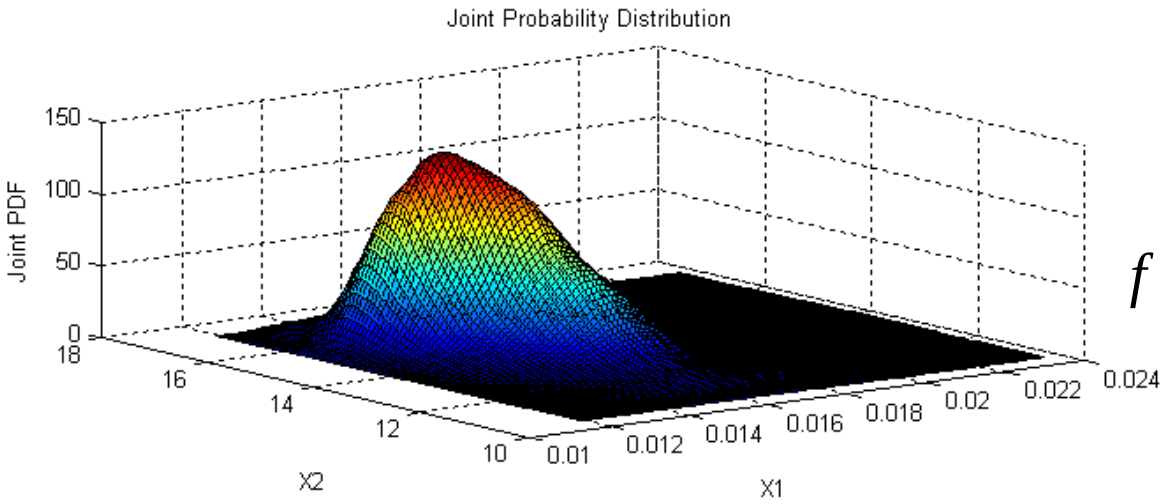
$F(X \mid Y)$: Conditional marginal cumulative distribution of $X \in \{X_1, X_2\}$ given that spill has occured at Y.

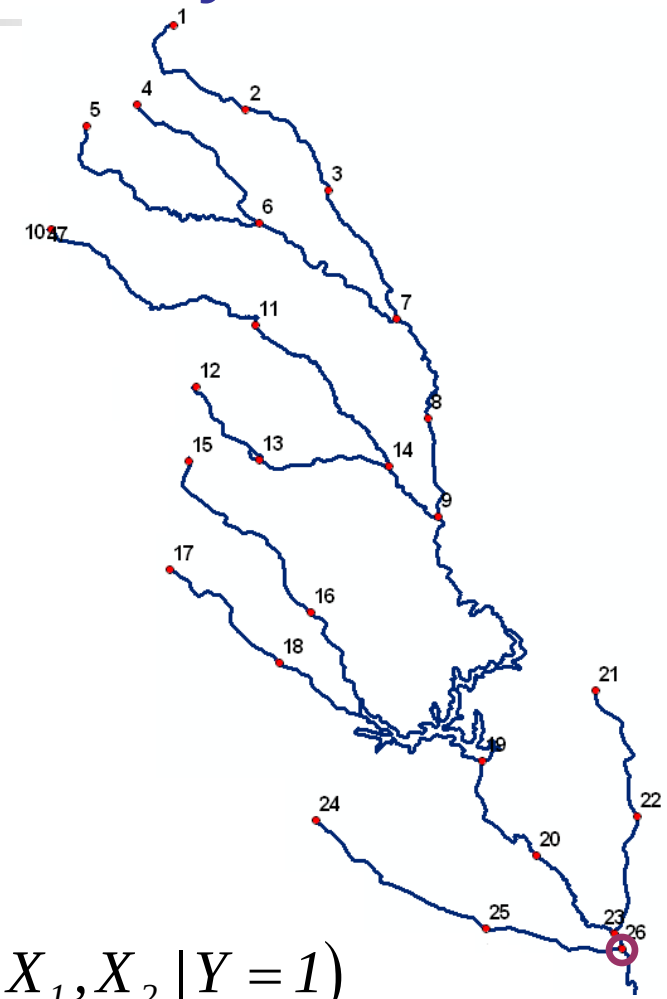
$f(X_1, X_2 \mid Y)$: Conditional joint probability density of X_1 and X_2 given that spill has occured at Y.

Conditional prior joint probability densities

For spills coming from Junction 1



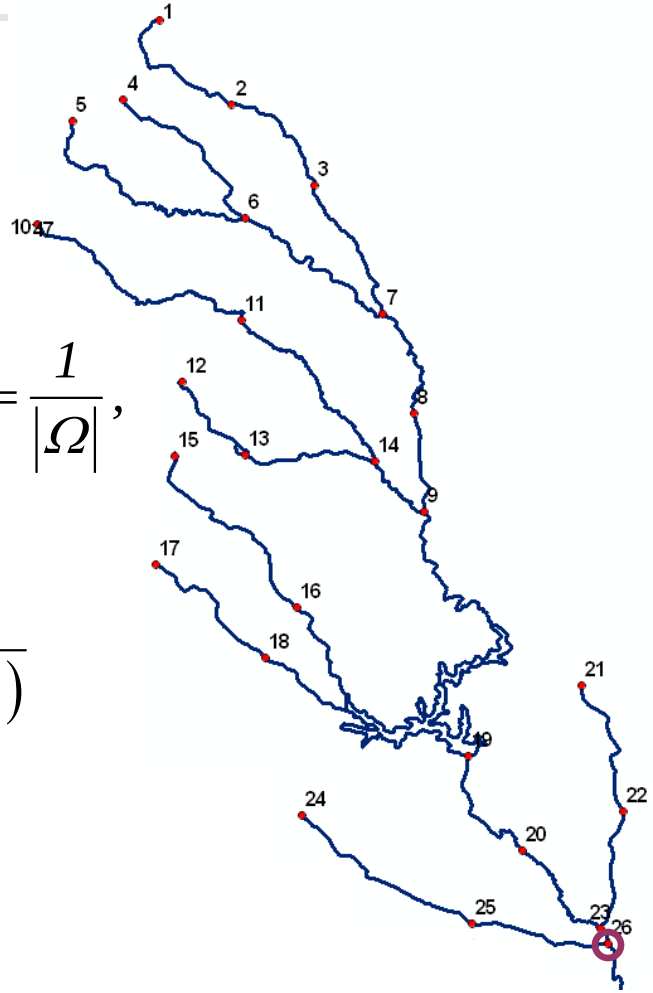




$$f\left(X_1, X_2 \mid Y = 1\right)$$

Conditional posterior joint probabilities

Bayes' Theorem:



Let $\Omega = \{1,2,...,M\}$ is the set of all classes (junctions) possible for a spill event. Then, for $j \in \Omega$ and $P(Y = j) = \frac{1}{|\Omega|}$,

$$P(Y = j \mid X_1 = x_1, X_2 = x_2) = \frac{f(X_1 = x_1, X_2 = x_2 \mid Y = j)}{\sum_{i \in \Omega} f(X_1 = x_1, X_2 = x_2 \mid Y = i)}$$

$P(Y \mid X_1, X_2)$: Conditional joint probability of $Y \in \Omega$ given that spill has features of X_1 and X_2.

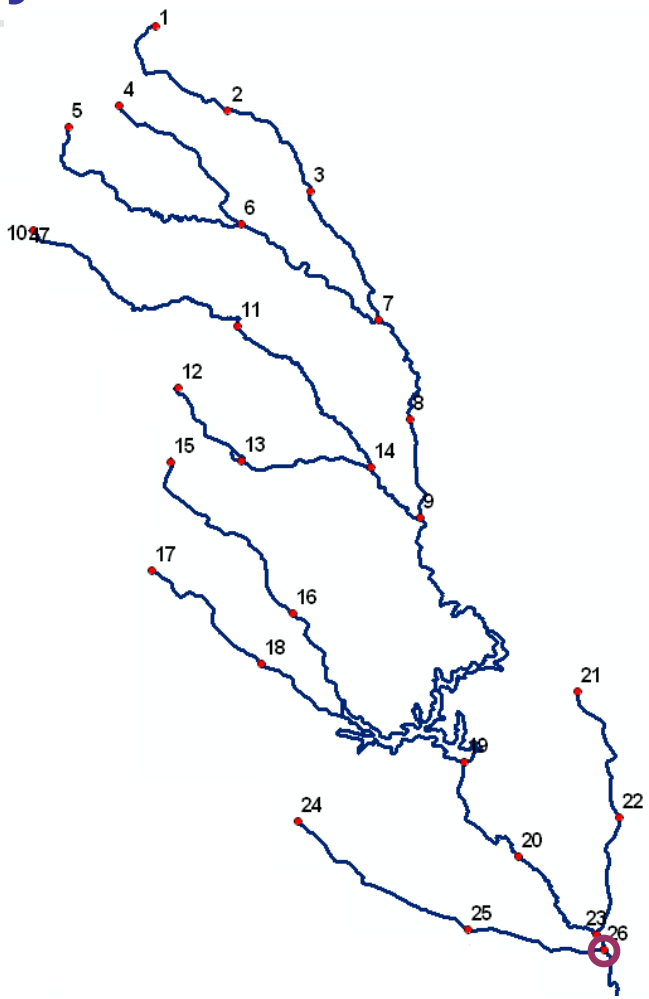
Summary of statistical analysis

Up to now, we can estimate three conditional probability values:

$P(Y \mid X_1)$: Conditional probability of $Y \in \Omega$ given that spill has feature of X_1

$P(Y \mid X_2)$: Conditional probability of $Y \in \Omega$ given that spill has feature of X_2

$P(Y \mid X_1, X_2)$: Conditional joint probability of $Y \in \Omega$ given that spill has features of X_1 and X_2.



How can we use this information to determine most possible classes, Y (or junctions) for a spill event having features x_1 and x_2 ?

Adaptive Sequential Feature Selection

Adaptive Feature Selection in Pattern Recognition and Ultra-Wideband Radar Signal Analysis; Phd Thesis by Hao Jiang ,CalTech, 2008.

Problem Statement:

Let (X,Y) be a pair of random variables: $X \in R^d$, $Y \in \{1,2,...M\}$

where M is the number of classes or patterns.

$D_n = \{(X^1,Y^1),(X^2,Y^2),...,(X^n,Y^n)\}$ is the tarining set with elements

being i.i.d. samples from a fixed but unknown distribution governing (X,Y).

$$X = [X_1 \quad X_2 \quad ... \quad X_d]^T$$

Assume that the classes Y take a prior distribution $\{P(Y=j), j=1,2,...,M\}$

The goal is to find a mapping g: $R^d \rightarrow \{1,2,...,M\}$ such that an arbitrary

unlabeled test data $x = [x_1 \quad x_2 \quad ... \quad x_d]^T \in R^d$ can be classified into

one of the M classes while optimizing some criterion.

Adaptive Sequential Feature Selection

Optimum Feature Selection:

Entropy corresponding to prior p.m.f:

$$H_0 = -\sum_{i=1}^{M} P(Y=i)\, log\left[P(Y=i)\right], \quad \text{to be unbiased } P(Y=j) = \frac{1}{M},\ j = 1,...,M$$

Class entropy conditioned on the sample x_i:

$$H_i = -\sum_{j=1}^{M} P(Y=j \mid X_i = x_i)\, log\left[P(Y=j \mid X_i = x_i)\right]$$

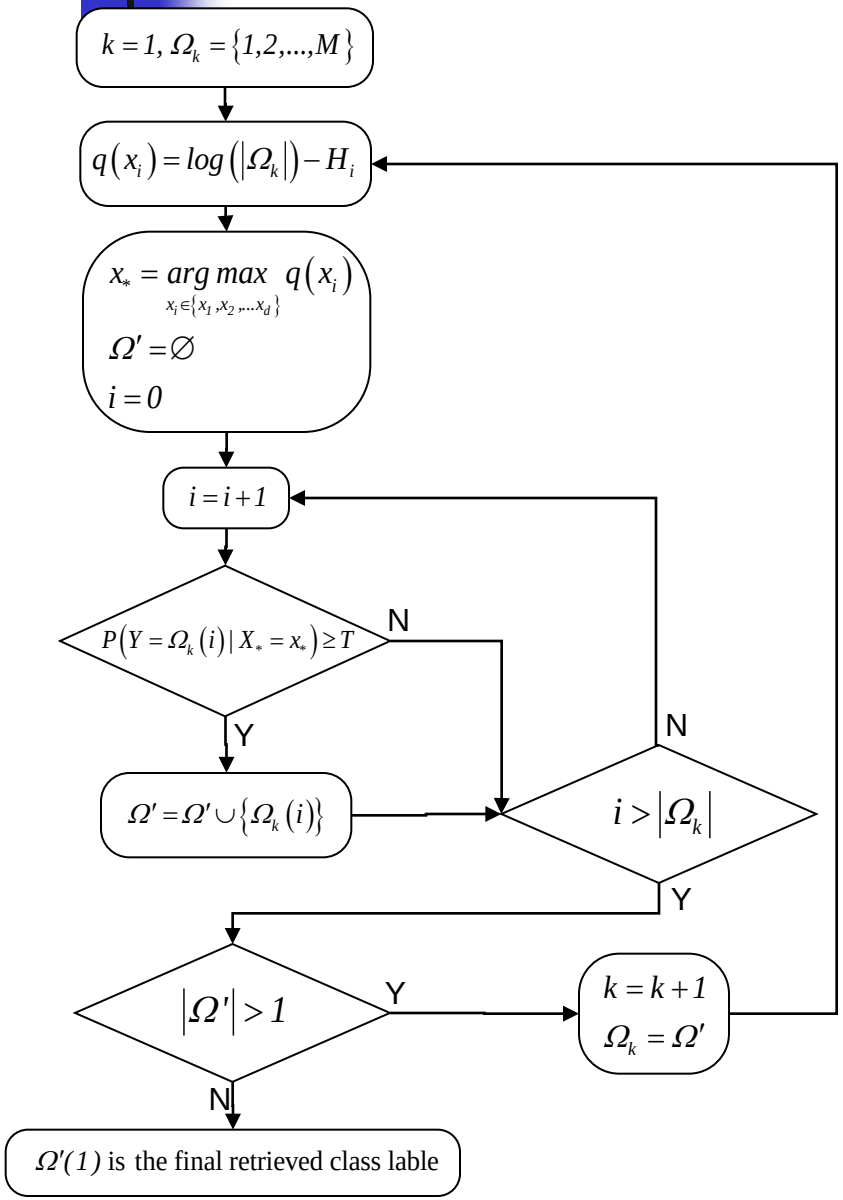
The quantity H_0-H_i measures how well the feature x_i (which is the ith component of the unlabeled test data x) reduces the complexity of the classification task.

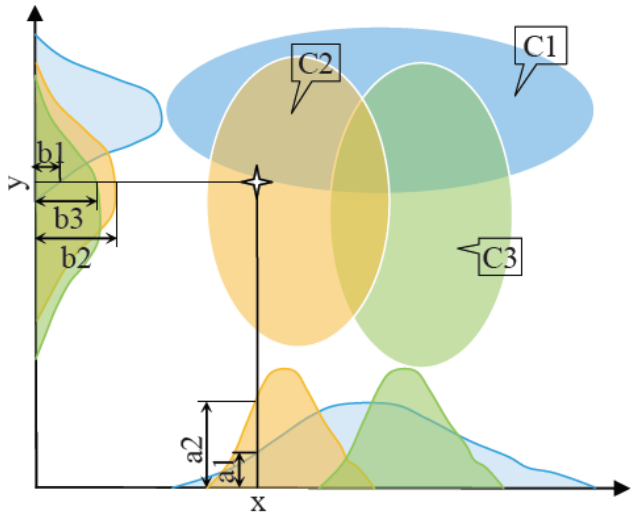
$$q(x_i) = H_0 - H_i$$

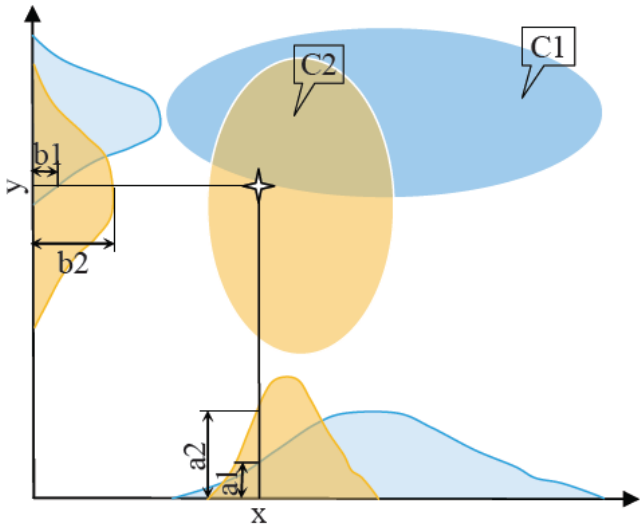
The best feature in terms of reducing the class entropy can be selected as:

$$x_* = \underset{x_i \in \{x_1,...,x_d\}}{\operatorname{argmax}}\ q(x_i)$$

Adaptive Sequential Feature Selection







Jiang (2008)

Adaptive Sequential Feature Selection

- Original form of ASFS algorithm reduces the initial class set to one final selected class by eliminating the classes using a probability threshold.

- However, final result is not always the correct answer.

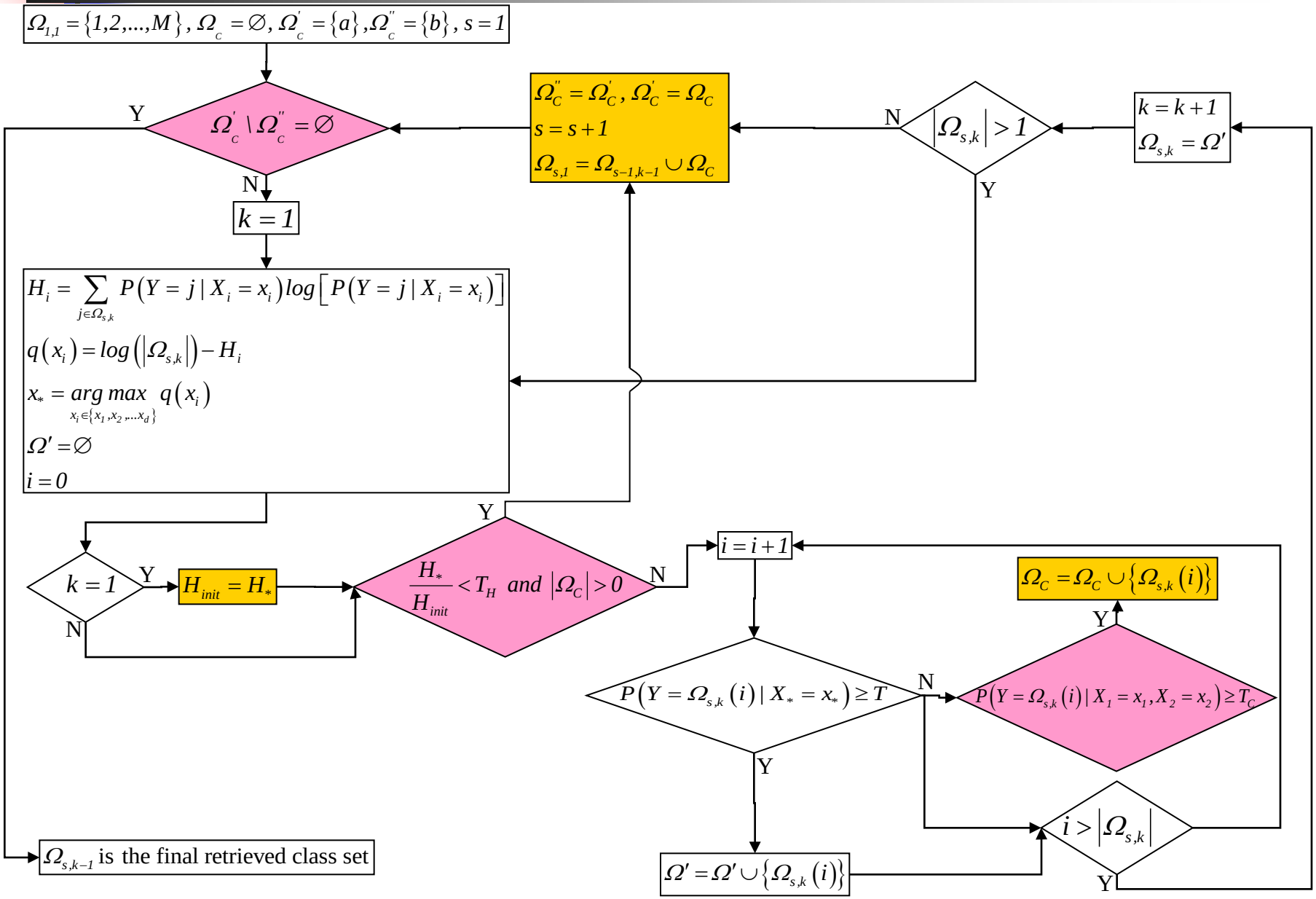
- We have made some modifications to add some flexibility to ASFS algorithm.
  - We introduced a copula set Ω_C in which we store some eliminated classes according to their performance against a joint probability threshold.

  - We determined a criterion to stop ASFS algorithm at some level where the correct class is in the final set.

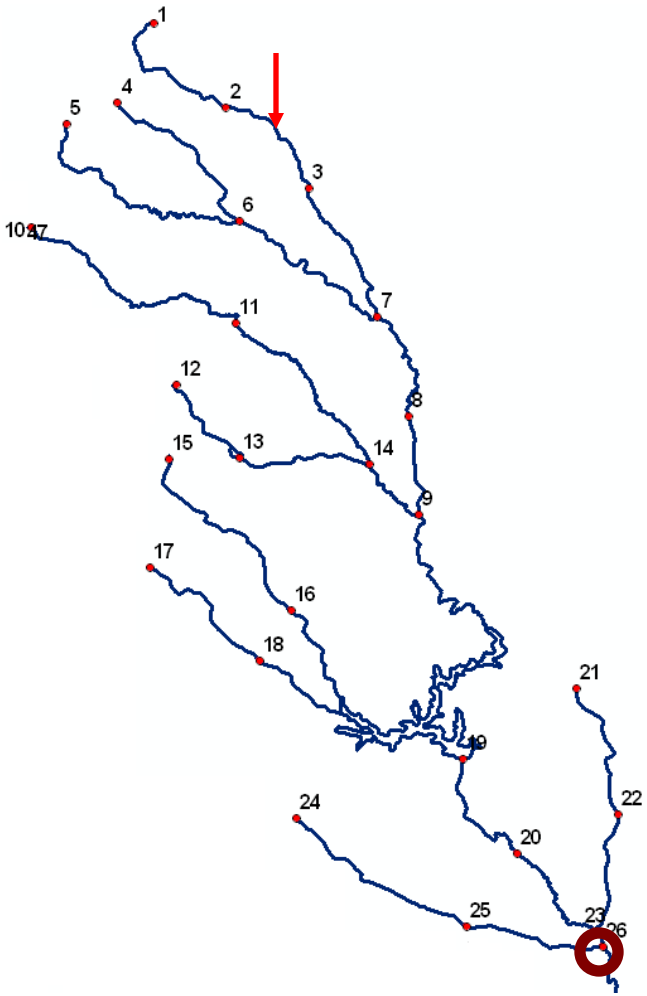
  - At the end of one ASFS cycle (according to our new stopping criteria), we added copula set into final class set obtained and re-ran ASFS algorithm.

  - This modified ASFS algorithm is stopped when two consecutive copula sets obtained are the same.

Modified ASFS



Results

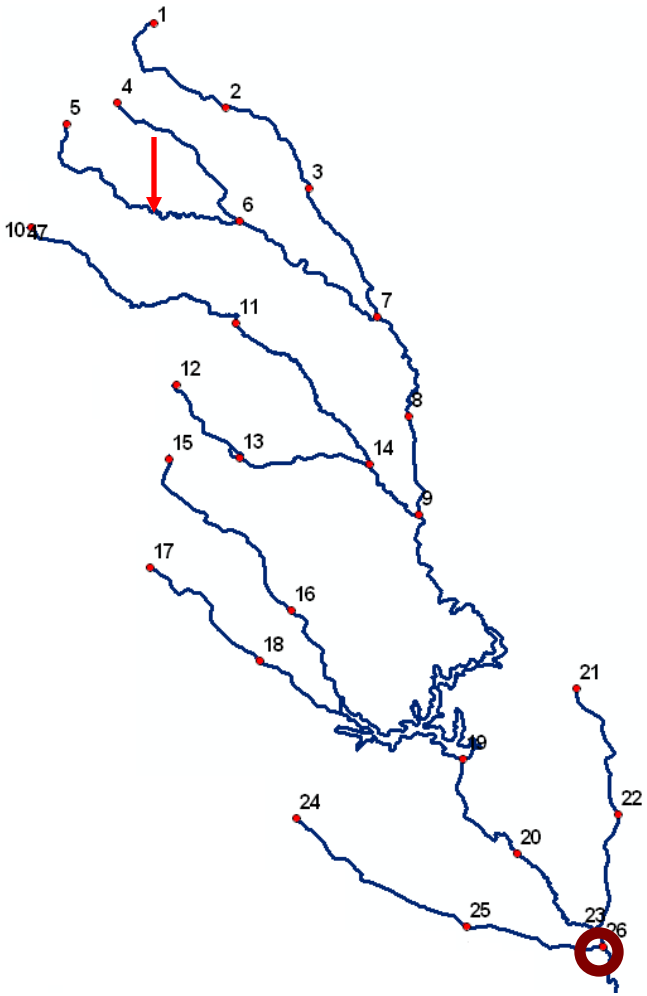


Spill_number:	1																	
Spill_between:	2	3																
FS	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	
Hr	1	1	1	1	1	1	1	1	1	1	1	1	1	0.99	0.96	0.93	0.89	0.85
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	4	6
	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	6	15
	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	6	15	16
	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	15	16	24
	7	7	7	7	7	7	7	7	7	7	7	7	15	15	15	16	24	
	8	8	8	8	8	10	11	11	11	13	13	15	16	16	16	24		
	9	9	9	10	10	11	12	13	13	14	15	16	17	18	24			
	10	10	10	11	11	12	13	14	14	15	16	17	18	24				
	11	11	11	12	12	13	14	15	15	16	17	18	24					
	12	12	12	13	13	14	15	16	16	17	18	24						
	13	13	13	14	14	15	16	17	17	18	24							
	14	14	14	15	15	16	17	18	18	24								
	15	15	15	16	16	17	18	21	24									
	16	16	16	17	17	18	21	24										
	17	17	17	18	18	21	24											
	18	18	18	21	21	24												
	19	21	21	24	24													
	20	22	24	25														
	21	24	25															
	22	25																
	23																	
	24																	
	25																	
	26																	

Spill_number:	1				
Spill_between:	2	3			
FS	P	P	P	P	P
Hr	1	0.97	0.93	0.9	0.86
	1	1	1	1	1
	2	2	2	2	2
	3	3	3	4	6
	4	4	4	6	15
	5	5	6	15	16
	6	6	15	16	24
	15	15	16	24	
	16	16	24		
	18	24			
	24				

Result_for_spill	1_wk	2_and	3		
	J	C	MM	Pe	Avg
	1	0.11	0.15	0.12	0.13
	2	0.25	0.2	0.19	0.21
	4	0.01	0.03	0.03	0.02
	6	0.22	0.22	0.18	0.21
	15	0.01	0.05	0.03	0.03
	16	0.15	0.17	0.17	0.17
	24	0.25	0.18	0.29	0.24

Results



Spill_number:	2																			
Spill_between:	5	6																		
FS	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	P	M	M	
Hr	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.98	0.89	0.86	0.76
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2
	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	3
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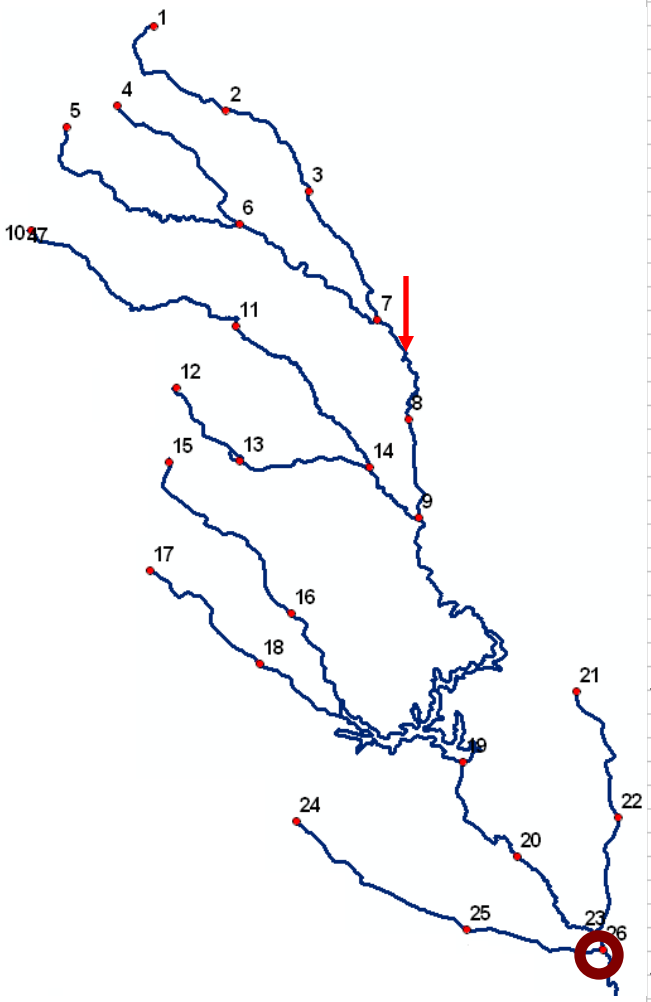
	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	6	6	6	6	7
	5	5	5	5	5	5	5	5	5	5	6	6	6	6	6	7	7	7	7	21
	6	6	6	6	6	6	6	6	6	6	7	7	7	7	7	8	8	16	21	24
	7	7	7	7	7	7	7	7	7	7	8	8	8	8	8	16	16	21	24	
	8	8	8	8	8	8	8	8	8	8	9	9	9	9	16	18	21	24		
	9	9	9	9	9	9	9	9	9	9	15	16	16	16	18	21	24			
	10	10	11	11	11	11	11	11	14	15	16	17	18	18	21	24				
	11	11	12	12	13	13	13	14	15	16	17	18	21	21	24					
	12	12	13	13	14	14	14	15	16	17	18	21	24	24						
	13	13	14	14	15	15	15	16	17	18	21	24	25							
	14	14	15	15	16	16	16	17	18	21	24	25								
	15	15	16	16	17	17	17	18	21	24	25									
	16	16	17	17	18	18	18	21	24	25										
	17	17	18	18	19	19	21	24	25											
	18	18	19	19	21	21	24	25												
	19	19	21	21	22	24	25													
	20	21	22	22	24	25														
	21	22	24	24	25															
	22	24	25	25																
	23	25	26																	
	24	26																		
	25																			
	26																			

Spill_number:	2				
Spill_between:	5	6			
FS	M	M	M	M	M
Hr	1	1	1	0.97	0.85
	1	1	1	1	2
	2	2	2	2	3
	3	3	3	3	6
	4	4	6	6	7
	5	6	7	7	21
	6	7	16	21	24
	7	16	21	24	
	16	21	24		
	21	24			
	24				

Result_for_spill	2_wh	5_and	6		
	J	C	MM	Pe	Avg
	1	0.04	0.01	0.13	0.06
	2	0.2	0.07	0.25	0.18
	3	0.09	0.19	0.1	0.13
	6	0.12	0.03	0.2	0.11
	7	0	0.43	0.01	0.15
	21	0	0.18	0	0.06
	24	0.55	0.1	0.32	0.32

Results

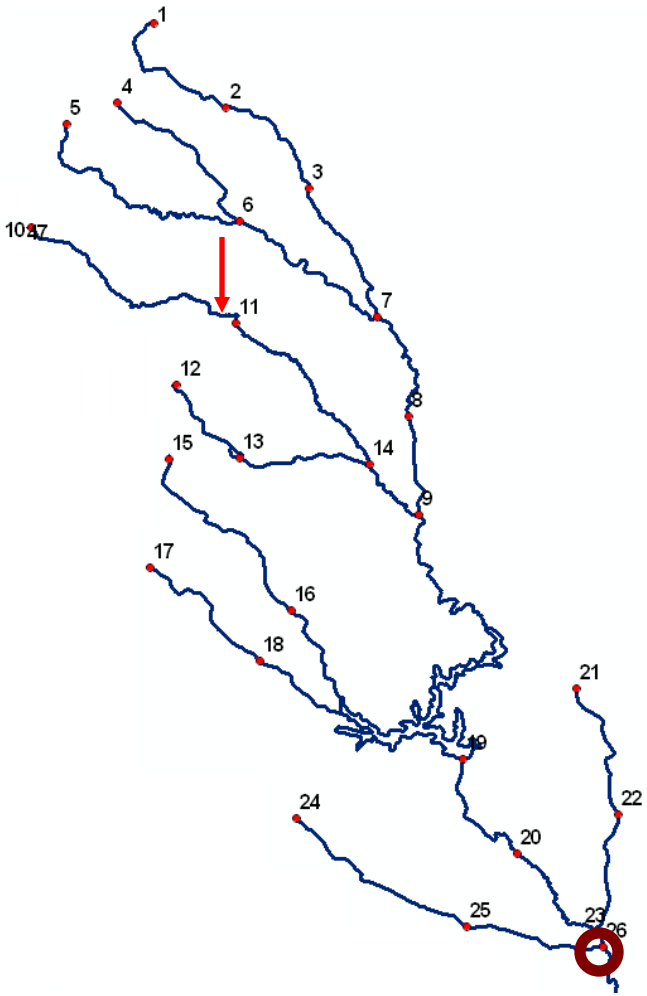


Spill_number:	3																				
Spill_between:	7	8																			
FS	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	M	M	M	
Hr	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.99	0.94	0.94	0.93	0.85
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	3	7	8
	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	3	3	3	7	8	9
	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	6	7	7	8	9	21
	4	4	4	4	4	4	4	4	4	4	6	6	6	6	6	7	8	8	9	21	25
	5	5	5	5	5	5	5	5	5	6	7	7	7	7	7	8	9	9	21	25	
	6	6	6	6	6	6	6	6	6	7	8	8	8	8	8	9	21	21	25		
	7	7	7	7	7	7	7	7	7	8	9	9	9	9	9	21	24	25			
	8	8	8	8	8	8	8	8	8	9	16	16	16	17	21	24	25				
	9	9	9	9	9	9	9	9	9	16	17	17	17	21	24	25					
	10	10	10	11	11	11	13	15	16	17	18	18	21	24	25						
	11	11	11	12	13	13	15	16	17	18	21	21	24	25							
	12	12	12	13	14	15	16	17	18	21	22	24	25								
	13	13	13	14	15	16	17	18	21	22	24	25									
	14	14	14	15	16	17	18	21	22	24	25										
	15	15	15	16	17	18	21	22	24	25											
	16	16	16	17	18	21	22	24	25												
	17	17	17	18	21	22	24	25													
	18	18	18	21	22	24	25														
	19	19	21	22	24	25															
	20	21	22	24	25																
	21	22	24	25																	
	22	24	25																		
	23	25																			
	24																				
	25																				
	26																				

Spill_number:	3		
Spill_between:	7	8	
FS	M	P	P
Hr	1	0.91	0.81
	7	8	8
	8	9	9
	9	21	25
	21	25	
	25		

Result_for_spill	3_wh	7_and	8		
	J	C	MM	Pe	Avg
	8	0.13	0.17	0.21	0.17
	9	0.42	0.38	0.41	0.41
	21	0.01	0.06	0.03	0.03
	25	0.44	0.39	0.35	0.39

Results

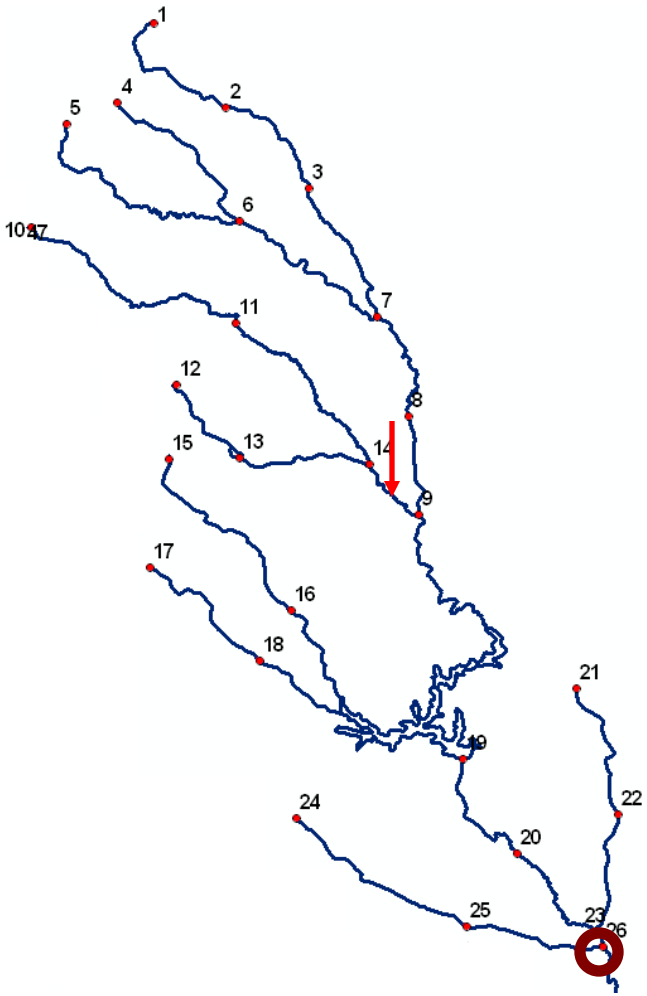


Spill_number:	4														
Spill_between:	10	11													
FS	P	P	P	P	P	P	P	P	P	P	P	P	P	P	
Hr	1	1	1	1	1	1	1	1	1	1	1	1	0.99	0.91	0.78
	1	1	1	1	1	1	1	1	4	4	4	10	10	11	11
	2	2	2	2	2	2	4	4	5	5	10	11	11	12	13
	3	3	3	4	4	4	5	5	10	10	11	12	12	13	14
	4	4	4	5	5	5	6	10	11	11	12	13	13	14	17
	5	5	5	6	6	6	10	11	12	12	13	14	14	17	
	6	6	6	10	10	10	11	12	13	13	14	17	17		
	7	7	10	11	11	11	12	13	14	14	17	18			
	8	10	11	12	12	12	13	14	15	17	18				
	9	11	12	13	13	13	14	15	17	18					
	10	12	13	14	14	14	15	17	18						
	11	13	14	15	15	15	17	18							
	12	14	15	16	16	17	18								
	13	15	16	17	17	18									
	14	16	17	18	18										
	15	17	18	24											
	16	18	24												
	17	24													
	18														
	19														
	20														
	21														
	22														
	23														
	24														
	25														
	26														

Spill_number:	4			
Spill_between:	10	11		
FS	P	M	P	
Hr	1	0.86	0.7	
	11	11	11	
	12	13	13	
	13	14	14	
	14	17		
	17			

Result_for_spill	4_wh	10_an	11		
	J	C	MM	Pe	Avg
	11	0.31	0.35	0.23	0.3
	13	0.32	0.34	0.25	0.3
	14	0.27	0.18	0.37	0.27
	17	0.1	0.13	0.15	0.13

Results

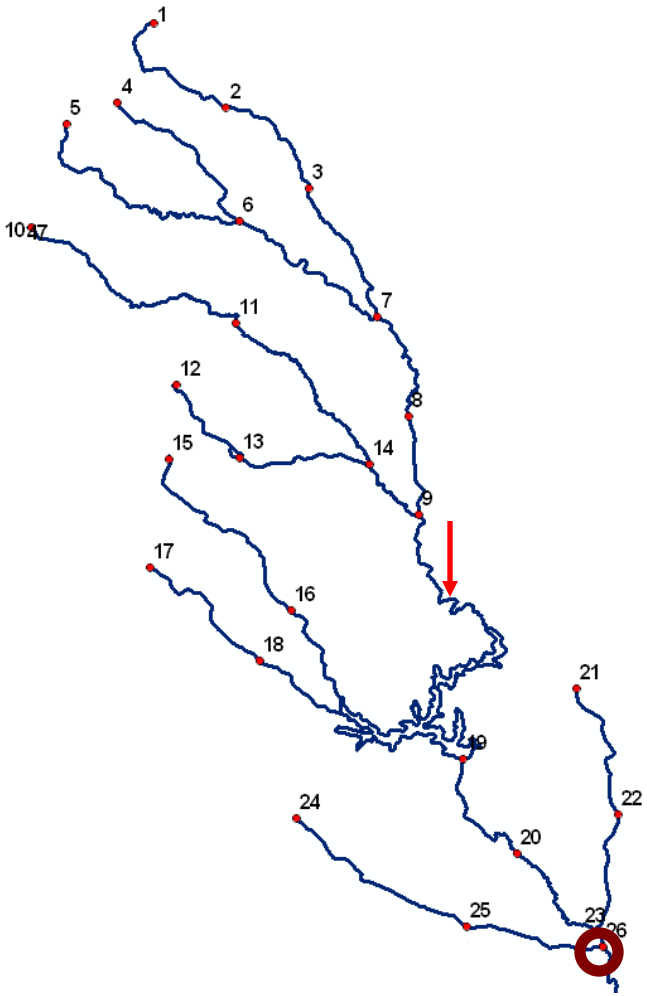


Spill_number:	5													
Spill_between:	14	9												
FS	P	P	P	P	P	P	P	P	P	P	M	P	M	
Hr	1	1	1	1	1	1	1	1	1	1	0.97	0.93	0.87	0.83
	1	1	1	1	1	1	1	1	4	4	4	4	11	11
	2	2	2	2	2	2	4	4	5	5	10	11	12	13
	3	3	3	4	4	4	5	5	10	10	11	12	13	14
	4	4	4	5	5	5	6	10	11	11	12	13	14	17
	5	5	5	6	6	6	10	11	12	12	13	14	17	18
	6	6	6	10	10	10	11	12	13	13	14	17	18	
	7	7	10	11	11	11	12	13	14	14	17	18		
	8	10	11	12	12	12	13	14	15	17	18			
	9	11	12	13	13	13	14	15	17	18				
	10	12	13	14	14	14	15	17	18					
	11	13	14	15	15	15	17	18						
	12	14	15	16	16	17	18							
	13	15	16	17	17	18								
	14	16	17	18	18									
	15	17	18	24										
	16	18	24											
	17	24												
	18													
	19													
	20													
	21													
	22													
	23													
	24													
	25													
	26													

Spill_number:	5			
Spill_between:	14	9		
FS	M	M	P	P
Hr	1	0.98	0.94	0.81
	10	11	11	11
	11	12	13	13
	12	13	14	14
	13	14	17	17
	14	17	18	
	17	18		
	18			

Result_for_spill	5_wh	14_an	9		
	J	C	MM	Pe	Avg
	11	0.1	0.09	0.17	0.12
	13	0.11	0.1	0.16	0.12
	14	0.43	0.28	0.29	0.33
	17	0.29	0.19	0.32	0.27
	18	0.08	0.34	0.06	0.16

Results

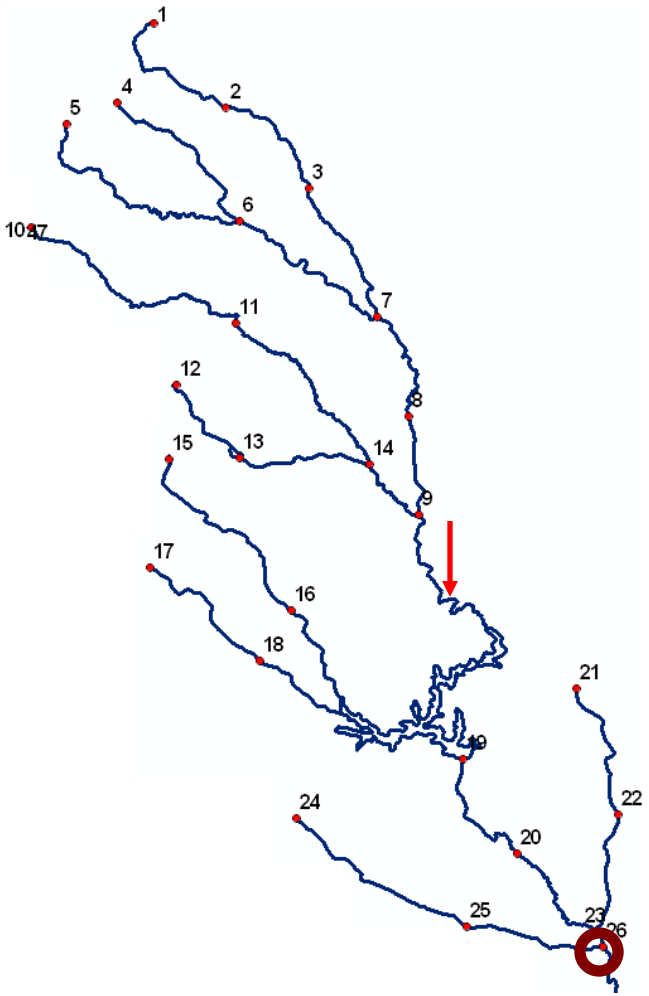


Spill_number:	6																	
Spill_between:	9	19																
FS	M	M	M	M	M	M	M	M	M	M	M	M	P	P	P	P		
Hr	1	1	1	1	1	1	1	1	1	1	1	1	0.84	0.84	0.83	0.81	0.73	
	1	1	1	1	1	1	1	1	1	1	1	2	2	2	3	7	7	
	2	2	2	2	2	2	2	2	2	2	2	3	3	3	7	8	8	
	3	3	3	3	3	3	3	3	3	3	3	7	7	7	8	9	9	
	4	4	4	4	4	5	5	7	7	7	7	8	8	8	9	21	25	
	5	5	5	5	5	6	7	8	8	8	8	9	9	9	21	25		
	6	6	6	6	6	7	8	9	9	9	9	21	21	21	25			
	7	7	7	7	7	8	9	17	18	19	21	22	24	25				
	8	8	8	8	8	9	17	18	19	21	22	24	25					
	9	9	9	9	9	17	18	19	21	22	24	25						
	10	15	15	16	17	18	19	21	22	24	25							
	11	16	16	17	18	19	21	22	24	25								
	12	17	17	18	19	21	22	24	25									
	13	18	18	19	21	22	24	25										
	14	19	19	21	22	24	25											
	15	21	21	22	24	25												
	16	22	22	24	25													
	17	24	24	25														
	18	25	25															
	19	26																
	20																	
	21																	
	22																	
	23																	
	24																	
	25																	
	26																	

Spill_number:	6			
Spill_between:	9	19		
FS	P	M	M	M
Hr	1	0.9	0.9	0.7
	7	7	8	9
	8	8	9	25
	9	9	25	
	21	25		
	25			

Result_for_spill	6_wh	9_and	19		
	J	C	MM	Pe	Avg
	8	0.02	0.06	0.07	0.05
	9	0.54	0.52	0.39	0.48
	25	0.45	0.42	0.54	0.47

Results

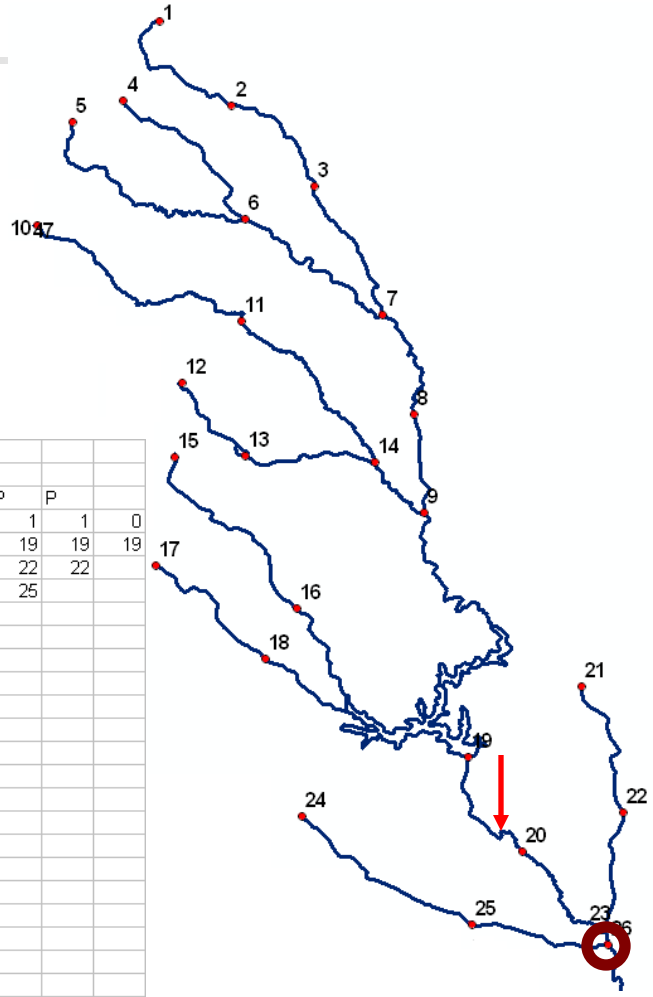


Spill_number:	7																	
Spill_between:	18	19																
FS	P	P	P	P	P	P	P	P	P	M	M	M	M	M	M	M	P	
Hr	1	1	1	1	1	1	1	1	1	0.94	0.94	0.94	0.94	0.93	0.91	0.89	0.86	0.83
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	6
	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	6	16
	5	5	5	5	5	5	5	5	5	5	5	5	5	5	6	6	16	18
	6	6	6	6	6	6	6	6	6	6	6	6	6	6	15	16	18	24
	7	7	7	7	7	7	7	7	7	11	11	14	15	15	16	18	24	
	8	8	8	8	8	10	11	11	11	13	14	15	16	16	18	24		
	9	9	9	10	10	11	12	12	13	14	15	16	17	18	24			
	10	10	10	11	11	12	13	13	14	15	16	17	18	24				
	11	11	11	12	12	13	14	14	15	16	17	18	24					
	12	12	12	13	13	14	15	15	16	17	18	24						
	13	13	13	14	14	15	16	16	17	18	24							
	14	14	14	15	15	16	17	17	18	24								
	15	15	15	16	16	17	18	18	24									
	16	16	16	17	17	18	21	24										
	17	17	17	18	18	21	24											
	18	18	18	21	21	24												
	19	21	21	24	24													
	20	22	24	25														
	21	24	25															
	22	25																
	23																	
	24																	
	25																	
	26																	

Spill_number:	7			
Spill_between:	18	19		
FS	M	P	M	P
Hr	1	0.97	0.93	0.86
	1	1	1	1
	2	2	2	2
	3	3	6	6
	5	6	16	16
	6	16	18	24
	16	18	24	
	18	24		
	24			

Result_for_spill	7_wh	18_an	19		
	J	C	MM	Pe	Avg
	1	0.1	0.1	0.14	0.11
	2	0.18	0.34	0.16	0.23
	6	0.16	0.16	0.2	0.17
	16	0.21	0.15	0.22	0.19
	18	0.04	0.06	0.03	0.04
	24	0.31	0.19	0.26	0.25

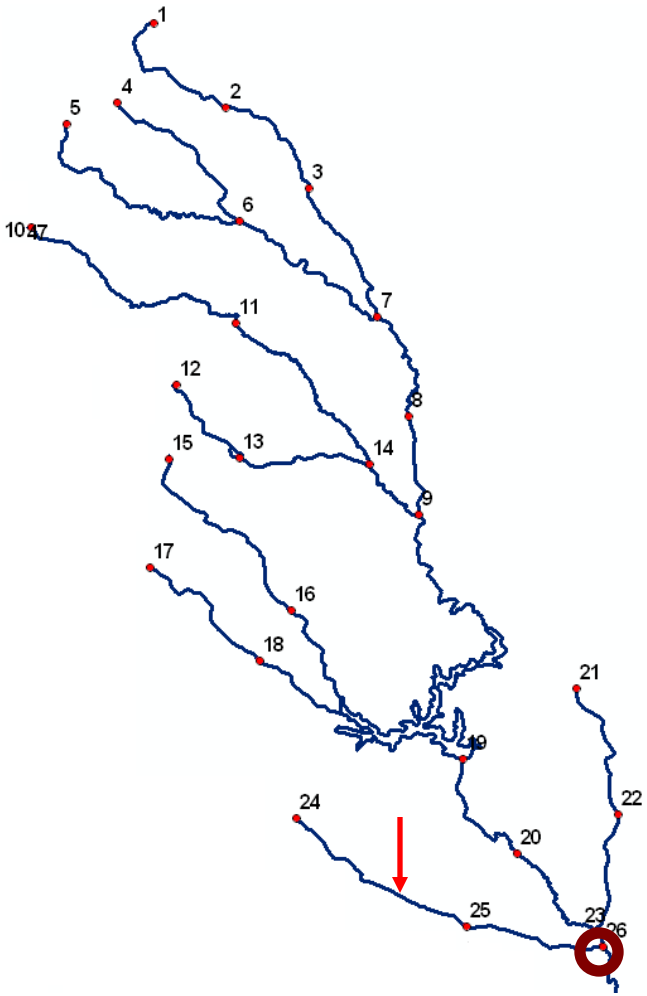
Results



Spill_number:	8																										
Spill_between:	19	20																									
FS	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	
Hr	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	8	8	8	19	19	19		
	2	2	2	2	2	2	2	2	2	2	2	2	2	2	3	3	7	7	8	9	9	19	22	22			
	3	3	3	3	3	3	3	3	3	3	3	3	3	3	7	7	8	8	9	17	19	22	25				
	4	4	4	4	4	4	4	4	4	4	4	4	6	7	8	8	9	9	17	19	22	25					
	5	5	5	5	5	5	5	5	5	5	5	6	7	8	9	9	17	17	19	22	25						
	6	6	6	6	6	6	6	6	6	6	6	7	8	9	17	17	18	19	22	25							
	7	7	7	7	7	7	7	7	7	7	7	8	9	17	18	18	19	22	25								
	8	8	8	8	8	8	8	8	8	8	8	9	17	18	19	19	22	25									
	9	9	9	9	9	9	9	9	9	9	9	17	18	19	21	22	25										
	10	10	10	11	11	11	13	15	16	17	17	18	19	21	22	25											
	11	11	11	12	12	13	15	16	17	18	18	19	21	22	25												
	12	12	12	13	13	15	16	17	18	19	19	21	22	25													
	13	13	13	14	15	16	17	18	19	21	21	22	25														
	14	14	14	15	16	17	18	19	21	22	22	25															
	15	15	15	16	17	18	19	21	22	24	25																
	16	16	16	17	18	19	21	22	24	25																	
	17	17	17	18	19	21	22	24	25																		
	18	18	18	19	21	22	24	25																			
	19	19	19	21	22	24	25																				
	20	20	21	22	24	25																					
	21	21	22	24	25																						
	22	22	24	25																							
	23	24	25																								
	24	25																									
	25																										
	26																										

Result_for_spill	8_wh	19_an	20		
	J	C	MM	Pe	Avg
	19	0.57	0.15	0.88	0.53
	22	0.43	0.85	0.12	0.47

Results

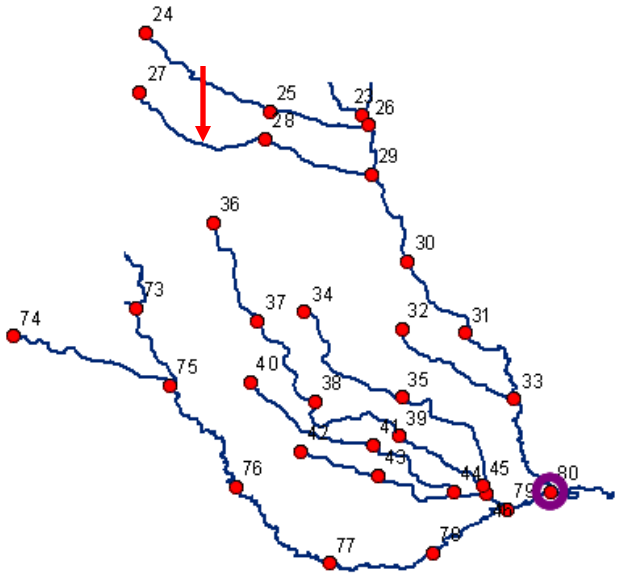


Spill_number:	9																					
Spill_between:	24	25																				
FS	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	M	
Hr	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.99	0.9	0.64
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	3	7	7	
	2	2	2	2	2	2	2	2	2	2	2	2	2	2	3	3	3	3	7	8	8	
	3	3	3	3	3	3	3	3	3	3	3	3	3	3	6	6	7	7	8	9	9	
	4	4	4	4	4	4	4	4	4	4	6	6	6	6	7	7	8	8	9	21	21	
	5	5	5	5	5	5	5	5	5	6	7	7	7	7	8	8	9	9	21	25		
	6	6	6	6	6	6	6	6	6	7	8	8	8	8	9	9	21	21	25			
	7	7	7	7	7	7	7	7	7	8	9	9	9	9	21	21	24	25				
	8	8	8	8	8	8	8	8	8	9	16	17	17	21	22	24	25					
	9	9	9	9	9	9	9	9	9	16	17	18	21	22	24	25						
	10	10	11	11	11	11	13	15	16	17	18	21	22	24	25							
	11	11	12	13	13	13	15	16	17	18	21	22	24	25								
	12	12	13	14	14	15	16	17	18	21	22	24	25									
	13	13	14	15	15	16	17	18	21	22	24	25										
	14	14	15	16	16	17	18	21	22	24	25											
	15	15	16	17	17	18	21	22	24	25												
	16	16	17	18	18	21	22	24	25													
	17	17	18	19	21	22	24	25														
	18	18	19	21	22	24	25															
	19	19	21	22	24	25																
	20	21	22	24	25																	
	21	22	24	25																		
	22	24	25																			
	23	25																				
	24																					
	25																					
	26																					

Spill_number:	9				
Spill_between:	24	25			
FS	P	P	M	P	
Hr	1	0.99	0.9	0.65	
	2	3	7	7	
	3	7	8	8	
	7	8	9	9	
	8	9	21	21	
	9	21	25		
	21	25			
	25				

Result_for_spill	9_wh	24_an	25		
	J	C	MM	Pe	Avg
	7	0.54	0.45	0.03	0.34
	8	0.36	0.35	0.1	0.27
	9	0	0	0.45	0.15
	21	0.11	0.2	0.02	0.11
	25	0	0	0.4	0.13

Results

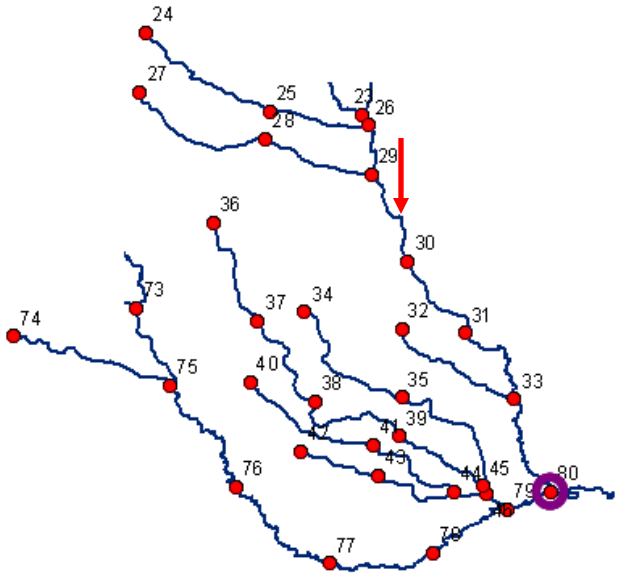


Spill_number	1															
Spill_betwee	27	28														
FS	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	
Hr	1	1	1	1	1	1	1	1	1	0.99	0.98	0.97	0.95	0.94	0.89	0.85
	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27
	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	35
	29	29	29	29	32	32	32	34	34	34	34	34	35	35	35	39
	30	30	32	32	34	34	34	35	35	35	35	35	37	38	39	40
	31	32	34	34	35	35	35	36	37	37	37	37	38	39	40	43
	32	34	35	35	36	36	36	37	38	38	38	38	39	40	43	45
	33	35	36	36	37	37	37	38	39	39	39	39	40	43	45	
	34	36	37	37	38	38	38	39	40	40	40	40	43	45		
	35	37	38	38	39	39	39	40	41	41	41	43	45			
	36	38	39	39	40	40	40	41	42	42	43	45				
	37	39	40	40	41	41	41	42	43	43	45					
	38	40	41	41	42	42	42	43	45	45						
	39	41	42	42	43	43	43	45	74							
	40	42	43	43	44	44	45	74								
	41	43	44	44	45	45	74									
	42	44	45	45	46	74										
	43	45	46	46	74											
	44	46	74	74												
	45	74	75													
	46	75														
	74															
	75															
	76															
	77															
	78															
	79															
	80															

Spill_number	1				
Spill_betwee	27	28			
FS	P	P	P	P	P
Hr	1	0.99	0.97	0.92	0.85
	27	27	27	27	27
	28	28	28	35	39
	35	35	35	39	40
	39	39	39	40	43
	40	40	40	43	45
	41	41	43	45	
	42	43	45		
	43	45			
	45				

Result	for	spill	numbe	1	which	is	betwe	27	and	28
	J	C	MM	Pe	Avg					
	27	0.065	0.09	0.077	0.077	←				
	35	0.028	0.05	0.046	0.043					
	39	0.231	0.28	0.17	0.227					
	40	0.095	0.15	0.11	0.118					
	43	0.203	0.15	0.241	0.199					
	45	0.378	0.28	0.355	0.337					

Results

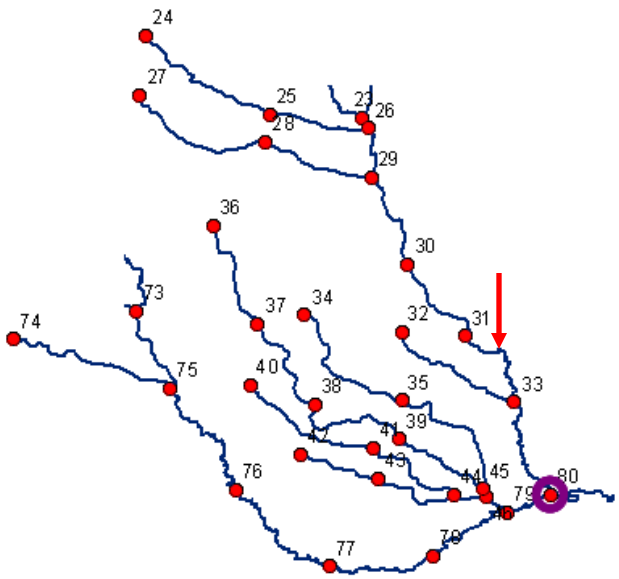


Spill_number	2																							
Spill_betwee	29	30																						
FS	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	P	M	M	
Hr	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.97	0.97	0.88	0.74
	27	27	27	27	27	27	27	27	27	27	27	28	28	28	28	28	28	28	28	29	29	29	29	29
	28	28	28	28	28	28	28	28	28	28	28	29	29	29	29	29	29	29	29	30	41	41	44	44
	29	29	29	29	29	29	29	29	29	29	29	30	30	30	30	30	30	30	30	41	43	44	46	75
	30	30	30	30	30	30	30	30	30	30	30	32	32	32	32	32	32	32	41	43	44	46	75	
	31	31	31	31	31	31	31	31	31	32	32	39	40	40	40	40	41	41	43	44	46	75		
	32	32	32	32	32	32	32	32	32	38	39	40	41	41	41	41	42	43	44	46	75			
	33	33	33	35	35	35	35	37	38	39	40	41	42	42	42	42	43	44	46	75				
	34	34	35	36	36	37	37	38	39	40	41	42	43	43	43	43	44	46	75					
	35	35	36	37	37	38	38	39	40	41	42	43	44	44	44	44	46	75						
	36	36	37	38	38	39	39	40	41	42	43	44	45	45	46	46	75							
	37	37	38	39	39	40	40	41	42	43	44	45	46	46	75	75								
	38	38	39	40	40	41	41	42	43	44	45	46	75	75	76									
	39	39	40	41	41	42	42	43	44	45	46	75	76	76										
	40	40	41	42	42	43	43	44	45	46	75	76	77											
	41	41	42	43	43	44	44	45	46	75	76	77												
	42	42	43	44	44	45	45	46	75	76	77													
	43	43	44	45	45	46	46	75	76	77														
	44	44	45	46	46	75	75	76	77															
	45	45	46	75	75	76	76	77																
	46	46	75	76	76	77	77																	
	74	75	76	77	77	80																		
	75	76	77	78	80																			
	76	77	78	80																				
	77	78	80																					
	78	80																						
	79																							
	80																							

Spill_number	2		
Spill_betwee	29	30	
FS	M	M	M
Hr	1	1	0.84
	28	29	29
	29	44	44
	44	46	75
	46	75	
	75		

Result	for	spill	numbe	2	which	is	betwe	29	and	30
	J	C	MM	Pe	Avg					
	29	0.389	0.37	0.323	0.361	←				
	44	0.166	0.26	0.182	0.202					
	46	0.084	0.07	0.164	0.106					
	75	0.362	0.3	0.331	0.332					

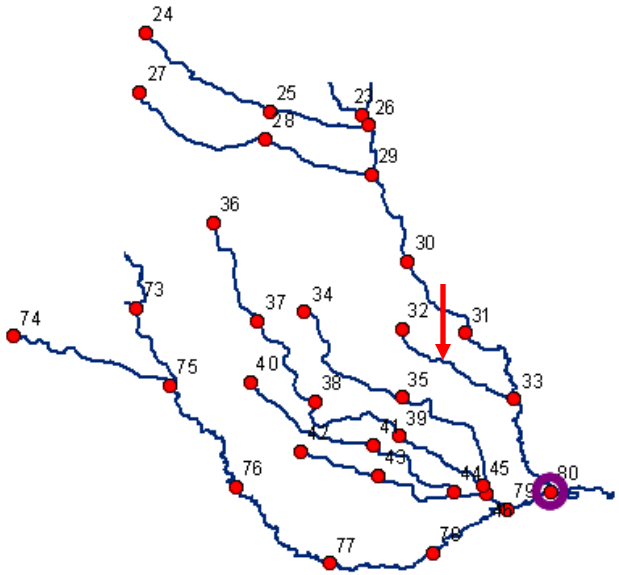
Results



Spill_number	3															
Spill_betwee	31	33														
FS	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	
Hr	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
	27	29	29	29	30	30	30	30	30	30	30	30	31	31	31	31
	28	30	30	30	31	31	31	31	31	31	31	31	46	46	77	
	29	31	31	31	33	33	33	33	33	33	33	46	76	77		
	30	33	33	33	41	43	43	43	44	46	46	76	77			
	31	40	41	41	43	44	44	44	46	76	76	77				
	32	41	42	43	44	46	46	46	76	77	77					
	33	42	43	44	46	75	75	76	77	78						
	34	43	44	46	75	76	76	77	78							
	35	44	46	75	76	77	77	78								
	36	46	75	76	77	78	78									
	37	75	76	77	78	80										
	38	76	77	78	80											
	39	77	78	80												
	40	78	80													
	41	80														
	42															
	43															
	44															
	45															
	46															
	74															
	75															
	76															
	77															
	78															
	79															
	80															

Result	for	spill	numbe	3	which	is	betwe	31	and	33						
	J	C	MM	Pe	Avg											
	31	0.745	0.69	0.683	0.705	←										
	77	0.255	0.31	0.317	0.295											

Results

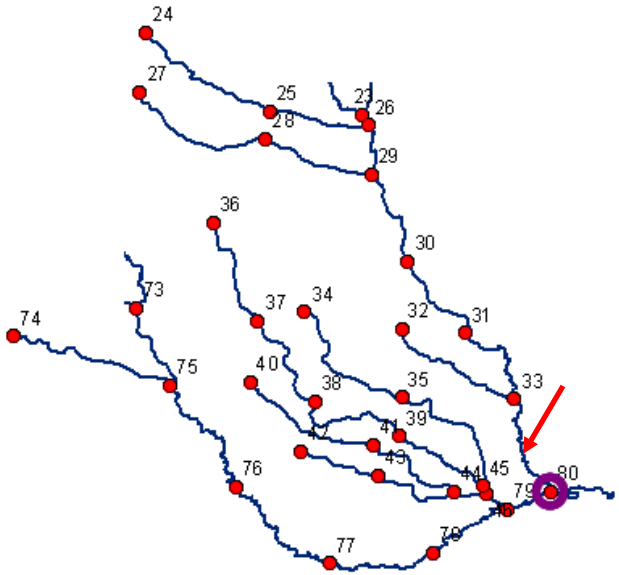


Spill_number	4																					
Spill_betwee	32	33																				
FS	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	P	M	M	P	P	P	
Hr	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.98	0.97	0.97	0.93	0.93	0.9	0.82
	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	28	28	28	28	28
	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	29	29	29	29	29
	29	29	29	29	29	29	29	29	29	29	29	29	29	29	29	29	29	32	32	32	32	32
	30	30	30	30	30	30	30	30	30	30	30	30	30	30	32	32	32	40	40	41	41	41
	31	31	31	31	31	31	31	32	32	32	32	32	32	32	39	40	40	41	41	43	43	44
	32	32	32	32	32	32	32	35	35	35	37	38	38	39	40	41	41	43	43	44	44	46
	33	33	34	34	34	35	35	36	37	37	38	39	39	40	41	42	43	44	44	46	46	
	34	34	35	35	35	36	36	37	38	38	39	40	40	41	42	43	44	45	46	75		
	35	35	36	36	36	37	37	38	39	39	40	41	41	42	43	44	45	46	75			
	36	36	37	37	37	38	38	39	40	40	41	42	42	43	44	45	46	75				
	37	37	38	38	38	39	39	40	41	41	42	43	43	44	45	46	75					
	38	38	39	39	39	40	40	41	42	42	43	44	44	45	46	75						
	39	39	40	40	40	41	41	42	43	43	44	45	45	46	75							
	40	40	41	41	41	42	42	43	44	44	45	46	46	75								
	41	41	42	42	42	43	43	44	45	45	46	75	75									
	42	42	43	43	43	44	44	45	46	46	75	76										
	43	43	44	44	44	45	45	46	75	75	76											
	44	44	45	45	45	46	46	75	76	76												
	45	45	46	46	46	75	75	76	77													
	46	46	74	74	75	76	76	77														
	74	74	75	75	76	77	77															
	75	75	76	76	77	80																
	76	76	77	77	80																	
	77	77	78	80																		
	78	78	80																			
	79	80																				
	80																					

Spill_number	4				
Spill_betwee	32	33			
FS	P	P	M	M	M
Hr	1	0.99	0.91	0.91	0.76
	28	28	28	28	28
	29	29	29	29	29
	32	32	32	32	41
	40	41	41	41	44
	41	43	44	44	
	43	44	46		
	44	46			
	46				

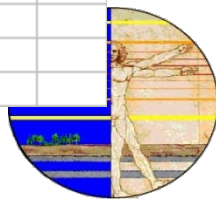
Result	for	spill	numbe	4	which	is	betwe	32	and	33
	J	C	MM	Pe	Avg					
	28	0.024	0.06	0.103	0.061					
	29	0.038	0.11	0.074	0.075					
	32	0.011	0.05	0.119	0.06	←				
	41	0.152	0.33	0.164	0.217					
	44	0.775	0.45	0.541	0.588					

Results

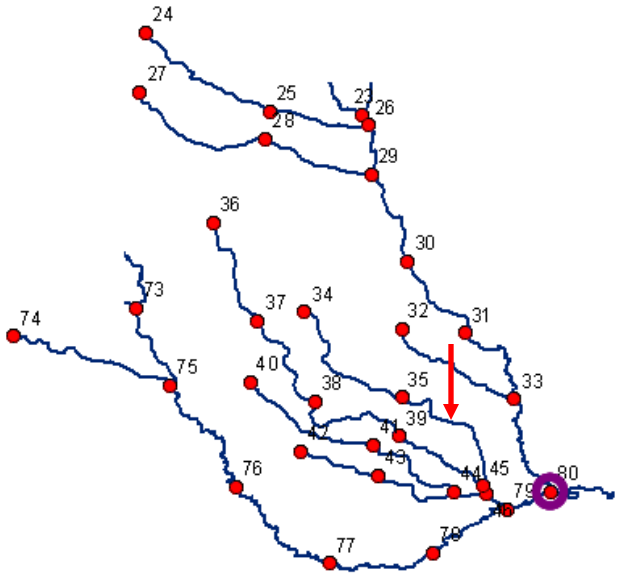


Spill_number	5									
Spill_betwee	33	80								
FS	M	M	M	P						
Hr	1	1	1	0	0					
	27	33	33	79	79					
	28	78	79	80						
	29	79	80							
	30	80								
	31									
	32									
	33									
	34									
	35									
	36									
	37									
	38									
	39									
	40									
	41									
	42									
	43									
	44									
	45									
	46									
	74									
	75									
	76									
	77									
	78									
	79									
	80									

Result	for	spill	numbe	5	which	is	betwe	33	and	80
	J	C	MM	Pe	Avg					
	79	0	0	1	0.5					
	80	0	1	0	0.5	←				



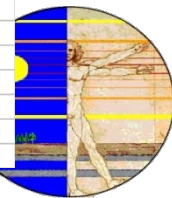
Results



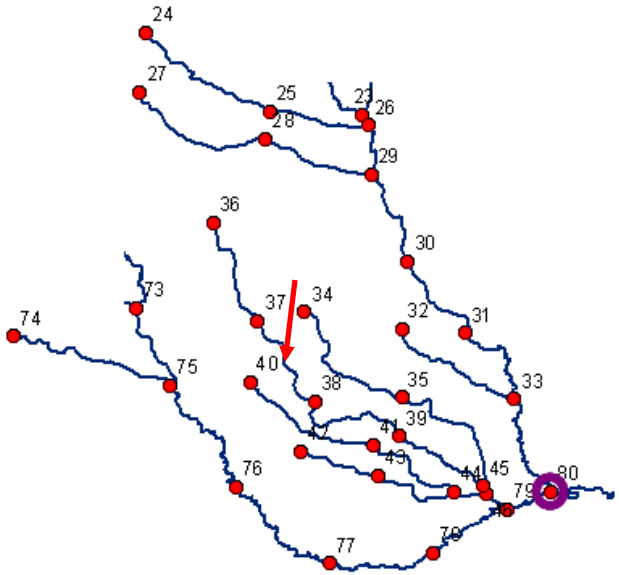
Spill_number	6												
Spill_betwee	35	45											
FS	P	P	P	P	P	P	P	P	P	P	P	P	P
Hr	1	1	1	1	1	1	1	1	1	1	0.98	0.96	0.89	0.81
	27	27	27	27	27	27	27	27	27	27	34	34	34
	28	28	28	28	34	34	34	34	34	34	35	35	35
	29	32	34	34	35	35	35	35	35	35	36	36	36
	30	34	35	35	36	36	36	36	36	36	37	37	37
	31	35	36	36	37	37	37	37	37	37	38	38	38
	32	36	37	37	38	38	38	38	38	38	42	42	
	33	37	38	38	39	39	39	40	42	42	74		
	34	38	39	39	40	40	40	42	43	74			
	35	39	40	40	41	42	42	43	74				
	36	40	41	41	42	43	43	74					
	37	41	42	42	43	45	74						
	38	42	43	43	45	74							
	39	43	44	45	74								
	40	44	45	74									
	41	45	74										
	42	74											
	43												
	44												
	45												
	46												
	74												
	75												
	76												
	77												
	78												
	79												
	80												

Spill_number	6		
Spill_betwee	35	45	
FS	P	P	P
Hr	1	0.93	0.84
	34	34	34
	35	35	35
	36	36	36
	37	37	37
	38	38	38
	42	42	
	74		

Result	for	spill	numbe	6	which	is	betwe	35	and	45
	J	C	MM	Pe	Avg					
	34	0.132	0.17	0.134	0.147					
	35	0.219	0.15	0.168	0.179	←				
	36	0.066	0.14	0.082	0.095					
	37	0.244	0.23	0.254	0.243					
	38	0.303	0.19	0.28	0.259					
	42	0.037	0.12	0.082	0.078					



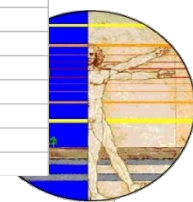
Results



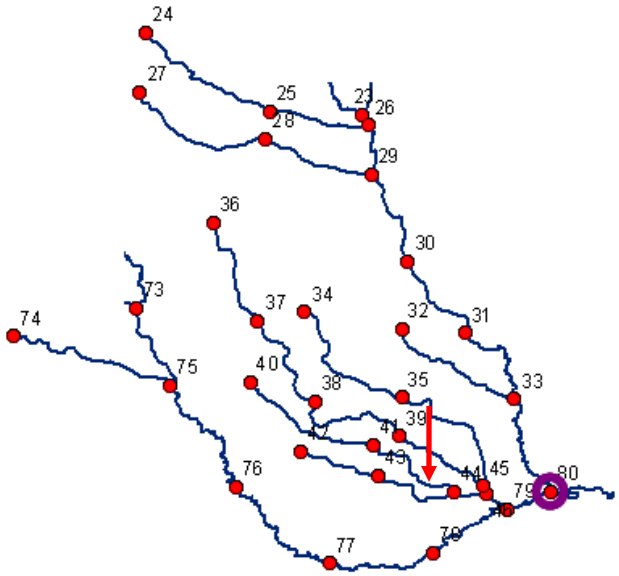
Spill_number	7										
Spill_betwee	37	38									
FS	P	P	P	P	P	P	P	M	P	M	
Hr	1	1	1	1	1	1	1	0.99	0.93	0.89	0.83
	27	27	27	27	27	27	27	27	27	27	27
	28	28	28	34	34	34	34	34	34	34	34
	29	34	34	35	35	35	35	35	35	35	35
	30	35	35	36	36	36	36	36	36	36	37
	31	36	36	37	37	37	37	37	37	37	38
	32	37	37	38	38	38	38	38	38	38	42
	33	38	38	39	39	39	40	42	42	42	
	34	39	39	40	40	40	42	43	43		
	35	40	40	41	42	42	43	74			
	36	41	41	42	43	43	74				
	37	42	42	43	45	74					
	38	43	43	45	74						
	39	44	45	74							
	40	45	74								
	41	74									
	42										
	43										
	44										
	45										
	46										
	74										
	75										
	76										
	77										
	78										
	79										
	80										

Spill_number	7			
Spill_betwee	37	38		
FS	M	M	P	P
Hr	1	0.98	0.91	0.83
	27	27	27	34
	34	34	34	35
	35	35	35	37
	36	36	37	38
	37	37	38	42
	38	38	42	
	42	42		
	74			

Result	for	spill	numbe	7	which	is	betwe	37	and	38
	J	C	MM	Pe	Avg					
	27	0.018	0.27	0.009	0.099					
	34	0.09	0.05	0.159	0.098					
	35	0.22	0.27	0.162	0.217					
	37	0.255	0.1	0.292	0.216	←				
	38	0.339	0.23	0.281	0.283	←				
	42	0.078	0.08	0.098	0.086					



Results

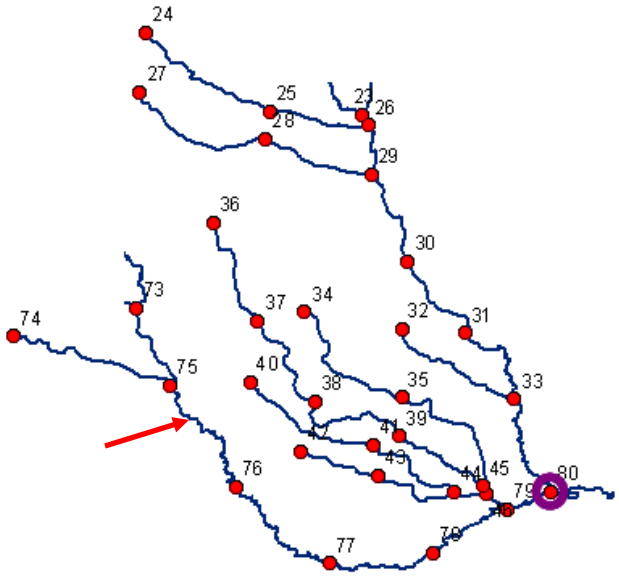


Spill_number	8																			
Spill_betwee	43	44																		
FS	P	P	P	P	P	P	M	M	M	M	M	M	M	M	M	P	M	M	M	
Hr	1	1	1	1	1	1	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.83	0.79	0.74	0.67	0.52
	27	27	27	27	27	27	27	27	27	27	27	27	27	27	28	28	28	28	28	28
	28	28	28	28	28	28	28	28	28	28	28	28	28	28	32	32	32	32	32	41
	29	29	29	29	29	29	32	32	32	32	32	32	32	32	40	40	40	40	41	43
	30	30	30	32	32	32	34	34	35	35	37	38	39	40	41	41	41	41	43	
	31	32	32	34	34	34	35	35	36	37	38	39	40	41	42	42	42	43		
	32	34	34	35	35	35	36	36	37	38	39	40	41	42	43	43	43			
	33	35	35	36	36	36	37	37	38	39	40	41	42	43	44	44				
	34	36	36	37	37	37	38	38	39	40	41	42	43	44	45					
	35	37	37	38	38	38	39	39	40	41	42	43	44	45						
	36	38	38	39	39	39	40	40	41	42	43	44	45							
	37	39	39	40	40	40	41	41	42	43	44	45								
	38	40	40	41	41	41	42	42	43	44	45									
	39	41	41	42	42	42	43	43	44	45										
	40	42	42	43	43	43	44	44	45											
	41	43	43	44	44	44	45	45												
	42	44	44	45	45	45	74													
	43	45	45	46	46	74														
	44	46	46	74	74															
	45	74	74	75																
	46	75	75																	
	74	76																		
	75																			
	76																			
	77																			
	78																			
	79																			
	80																			

Spill_number	8	
Spill_betwee	43	44
FS	M	P
Hr	1	0.76
	28	28
	32	41
	41	43
	43	

Result	for	spill	numbe	8	which	is	betwe	43	and	44
	J	C	MM	Pe	Avg					
	28	0.129	0.11	0.27	0.169					
	32	0.191	0.1	0.412	0.233					
	41	0.565	0.65	0.266	0.493					
	43	0.115	0.15	0.053	0.105	←				

Results

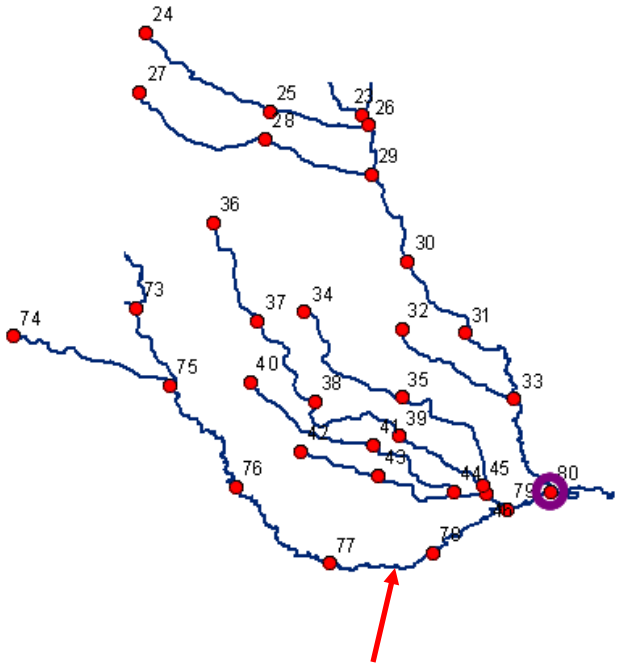


Spill_number	9																						
Spill_betwee	75	76																					
FS	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	
Hr	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.93	0.8
	27	27	27	27	27	27	27	27	27	27	27	28	28	28	28	28	28	28	28	29	29	29	30
	28	28	28	28	28	28	28	28	28	28	28	29	29	29	29	29	29	29	29	30	30	30	46
	29	29	29	29	29	29	29	29	29	29	29	30	30	30	30	30	30	30	30	32	44	46	75
	30	30	30	30	30	30	30	30	30	30	30	32	32	32	32	32	32	32	32	44	46	75	76
	31	31	31	31	31	31	31	31	31	31	32	35	39	40	40	40	41	41	44	46	75	76	
	32	32	32	32	32	32	32	32	32	32	35	39	40	41	41	41	43	44	46	75	76		
	33	33	34	34	35	35	35	35	35	35	39	40	41	43	43	43	44	46	75	76			
	34	34	35	35	36	37	37	38	38	39	40	41	43	44	44	44	46	75	76				
	35	35	36	36	37	38	38	39	39	40	41	43	44	45	46	46	75	76					
	36	36	37	37	38	39	39	40	40	41	43	44	45	46	75	75	76						
	37	37	38	38	39	40	40	41	41	43	44	45	46	75	76	76							
	38	38	39	39	40	41	41	42	43	44	45	46	75	76	77								
	39	39	40	40	41	42	42	43	44	45	46	75	76	77									
	40	40	41	41	42	43	43	44	45	46	75	76	77										
	41	41	42	42	43	44	44	45	46	75	76	77											
	42	42	43	43	44	45	45	46	75	76	77												
	43	43	44	44	45	46	46	75	76	77													
	44	44	45	45	46	74	75	76	77														
	45	45	46	46	74	75	76	77															
	46	46	74	74	75	76	77																
	74	74	75	75	76	77																	
	75	75	76	76	77																		
	76	76	77	77																			
	77	77	78																				
	78	78																					
	79																						
	80																						

Spill_number	9		
Spill_betwee	75	76	
FS	P	P	P
Hr	1	0.93	0.8
	29	29	30
	30	30	46
	44	46	75
	46	75	76
	75	76	
	76		

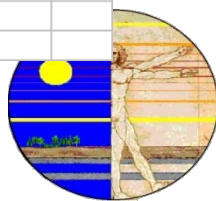
Result	for	spill	numbe	9	which	is	betwe	75	and	76
	J	C	MM	Pe	Avg					
	29	0.143	0.15	0.046	0.113					
	30	0.402	0.28	0.308	0.331					
	46	0.125	0.25	0.192	0.188					
	75	0.33	0.3	0.071	0.234	←				
	76	0	0.02	0.383	0.134	←				

Results



Spill_number	10														
Spill_betweer	77	78													
FS	M	M	M	M	M	M	M	M	M	M	M	M	M	M	
Hr	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
	27	29	29	30	30	30	30	30	30	31	31	31	31	31	31
	28	30	30	31	31	31	31	31	31	33	33	33	33	77	
	29	31	31	33	33	33	33	33	33	46	46	77	77		
	30	33	33	41	41	43	44	44	46	76	77	78			
	31	41	41	43	43	44	46	46	76	77	78				
	32	42	43	44	44	46	76	76	77	78					
	33	43	44	46	46	76	77	77	78						
	34	44	46	75	76	77	78	78							
	35	46	75	76	77	78	80								
	36	75	76	77	78	80									
	37	76	77	78	80										
	38	77	78	80											
	39	78	80												
	40	80													
	41														
	42														
	43														
	44														
	45														
	46														
	74														
	75														
	76														
	77														
	78														
	79														
	80														

Result	for	spill	numbe	10	which	is	betwe	77	and	78					
	J	C	MM	Pe	Avg										
	31	0.954	0.93	0.699	0.86										
	77	0.046	0.07	0.301	0.14	←									



Conclusions

- Modified ASFS algorithm performed well for both regions tested by total of 20 spill events.
    - The correct answer was kept in final sets for all 20 spill events.
    - The worst performance was reducing initial class set of 26 elements to a final selected set of 7 elements.
    - The best performance was reducing the initial class set of 27 elements to a final selected set of 2 elements.
- Final set is ranked according to the average of posterior conditional probabilities obtained for each single feature and joint behavior of the two.
- Once the statistical data is obtained from simulations (takes about 2 weeks), this method is fast (takes about 30 seconds to analyze all 20 spills).

Thank you…