Strategies for Sustainable Development Planning of Savanna System Using Optimal Control Model

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Introduction

Savanna system

- A savanna system is a complex natural ecosystem that describes competition behaviors between grasses and woody vegetation under driven of grazing cattle in a semi-arid rangeland

Introduction

Sustainable development

- The development that "meets the needs of the present without compromising the ability of future generations to meet their own needs" by Brundtland Commission (*Brundtland*, 1987)

- Associated with economy, sociology, and environment

- Junction of profit between present and future

Introduction

Sustainable development of savanna system

- meets the increasing demand for present economic profit by raising cattle
- protects ecological environment of rangeland to guarantee promoting needs of future development
- development sustainability = resilience

Introduction

Ecological resilience

- The capacity of a system to absorb disturbance and reorganize while undergoing change so as to still retain essentially the same function, structure, identity, and feedbacks (*Walker et al.*, 2004)

Introduction

Resilience of savanna system

- Resilience of savanna system depends on current state and “disturbance”

  - Current state is a result from previous management strategies and cannot be changed

  - Stocking rate is an external “disturbance” which reflects philosophy and targets of new management strategies and is controllable.

Introduction

Research on savanna system

- Resilience and stability (*Walker*, *Ludwig et al.*).
- Effects of physical, ecological, and economic factor on resilience (*Anderies et al.*).
- Stochastic model for rainfall uncertainty to find robust management strategies (*Janssen et al.*).

Introduction

Sustainable development planning

- Resilience thinking: a perspective for analysis of social-ecological systems

- Optimization for conservation: an outcome-oriented tool to obtain a defensible solution to a well-defined problem (*Fisher et al.)*

- Integration of resilience thinking and optimization for conservation to find strategy for sustainable development planning

Stability Analysis

Savanna system

$$\begin{cases} \dfrac{dg(t)}{dt} = r_g g\left(1 - s - c_{gg} g - c_{wg} w\right) \\ \dfrac{dw(t)}{dt} = r_w \left[\alpha + w\left(1 - c_{gw} g - c_{ww} w\right)\right] \end{cases}$$

g: density of grass

w: density of woody vegetation

s: stocking rate of cattle

others: parameters to describe characteristics of rangeland

Stability Analysis

Two stable equilibrium states

- Absolute stable state: (g*, w*) = (0, 1)
- Conditional stable state:

$$(g^*, w^*) = \left(0.703 - 0.312s + 0.558\sqrt{4s^2 - 5.2s + 1.583},\right.$$
$$\left. 0.508 - 0.782s - 0.391\sqrt{4s^2 - 5.2s + 1.583}\right)$$

Stability Analysis

Two stable equilibrium states

- Absolute stable state
  - Resilience = 0
  - Development sustainability = 0

- Conditional stable state
  - Resilience = f(s)
  - $s < 0.486$ (constant stocking rate)

Stability Analysis

Case: s = 0.25

$r_g = 1.5$

$r_w = 1$

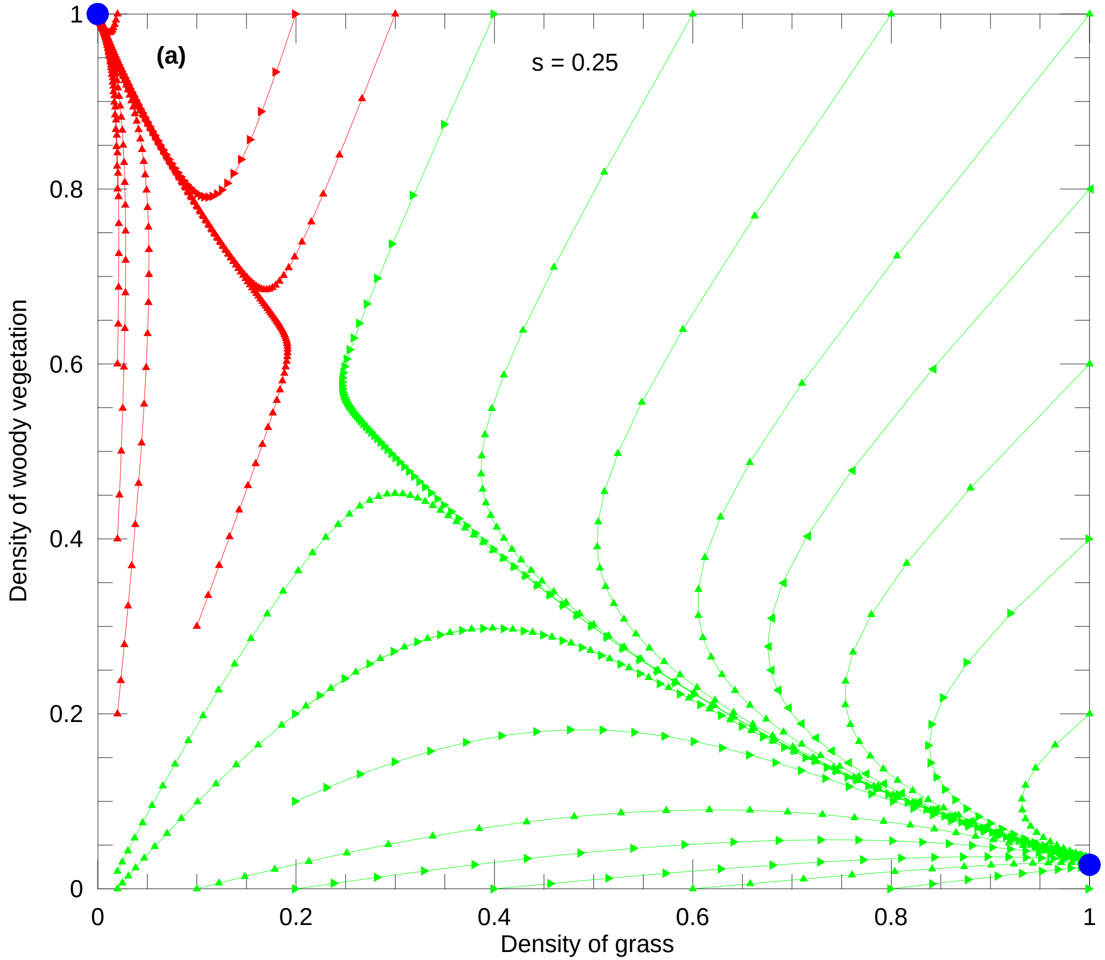
$c_{gg} = 0.7$

$c_{wg} = 1$

$c_{gw} = 2$

$c_{ww} = 1.03$

$\alpha = 0.03$



Stability Analysis

Case: s = 0.45

$r_g = 1.5$

$r_w = 1$

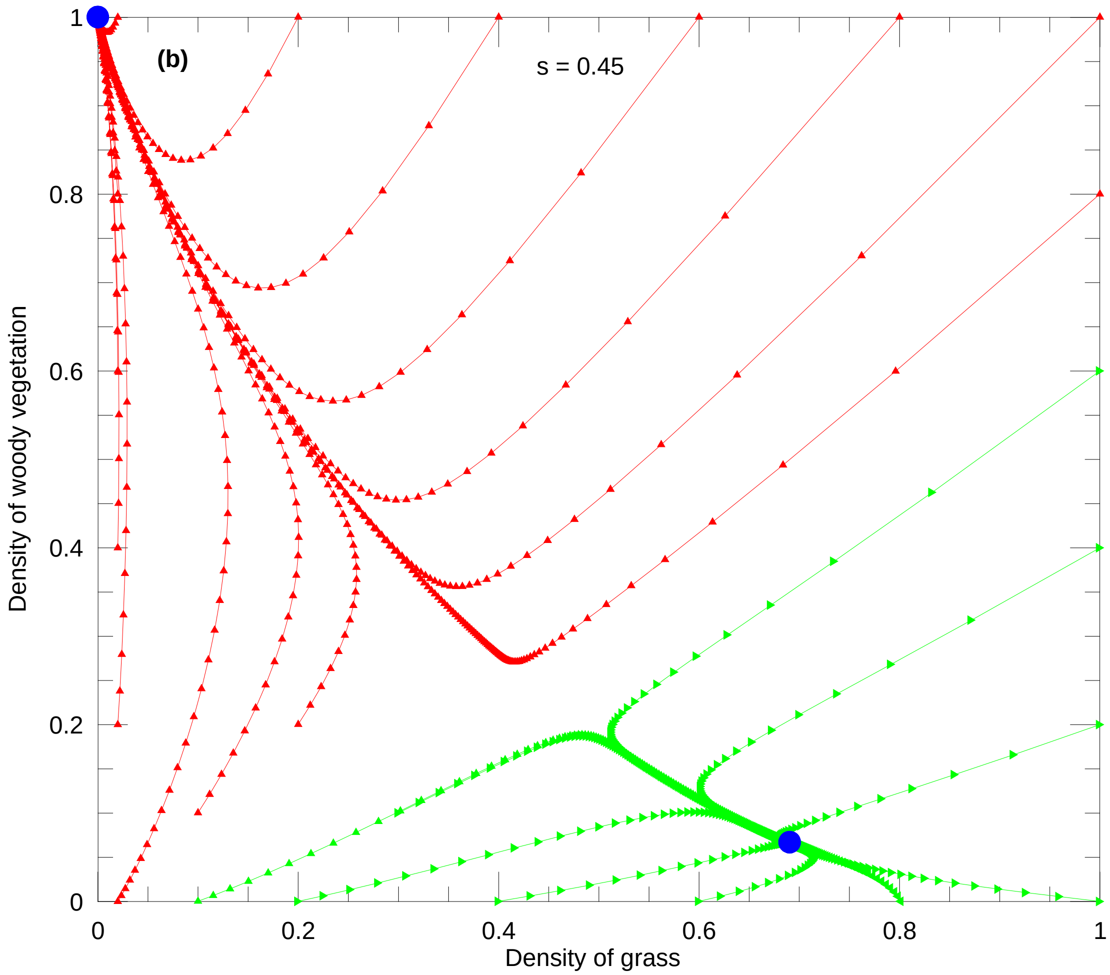
$c_{gg} = 0.7$

$c_{wg} = 1$

$c_{gw} = 2$

$c_{ww} = 1.03$

$\alpha = 0.03$



Stability Analysis

Observations

- Which stable point grass and woody vegetation approach is dependent on initial states and stocking rate
- Stable region with resilience depends on stocking rate. It shrinks as stocking rate increases and will disappear when $s > 0.486$.

Model and Algorithm

Sustainable development planning

- Control the system operation in the area close to second stable point
- Obtain maximum profit for development of grazing cattle without loss of ecological resilience of the system

Model and Algorithm

Optimization model

$$J = miaximize\left\{\frac{1}{2}\int_{t_0}^{t_f} rs^2(t)dt\right\}$$

$$\begin{cases} \dfrac{dg(t)}{dt} = r_g g\left(1 - s - c_{gg}g - c_{wg}w\right) \\ \dfrac{dw(t)}{dt} = r_w\left[\alpha + w\left(1 - c_{gw}g - c_{ww}w\right)\right] \end{cases}$$

$$X(t_0) = \left(g_0, w_0\right)^T$$

$$X(t_f) = \left(g_f, w_f\right)^T$$

Model and Algorithm

Optimal control model

$$J = miaximize\left\{\frac{1}{2}\int_{t_0}^{t_f} rs^2(t)dt - \frac{1}{2}h_g\left[g(t_f) - g^e(t_f)\right]^2 - \frac{1}{2}h_w\left[w(t_f) - w^e(t_f)\right]^2\right\}$$

$$\begin{cases} \dfrac{dg(t)}{dt} = r_g g\left(1 - s - c_{gg} g - c_{wg} w\right) \\ \dfrac{dw(t)}{dt} = r_w\left[\alpha + w\left(1 - c_{gw} g - c_{ww} w\right)\right] \end{cases}$$

$$X(t_0) = \left(g_0, w_0\right)^T$$

Model and Algorithm

Pontryagin's optimal principle

- **Hamiltonian function**

$$H(\mathbf{X}(t), s(t), \boldsymbol{\lambda}(t), t) = \frac{1}{2} r s^2 + \lambda_1 \left\{ r_g g \left(1 - s - c_{gg} g - c_{wg} w \right) \right\} + \lambda_2 \left\{ r_w \left(a + w \left(1 - c_{gw} g - c_{ww} w \right) \right) \right\}$$

Model and Algorithm

Pontryagin's optimal principle

- **Necessary conditions for s* to be optimal control**

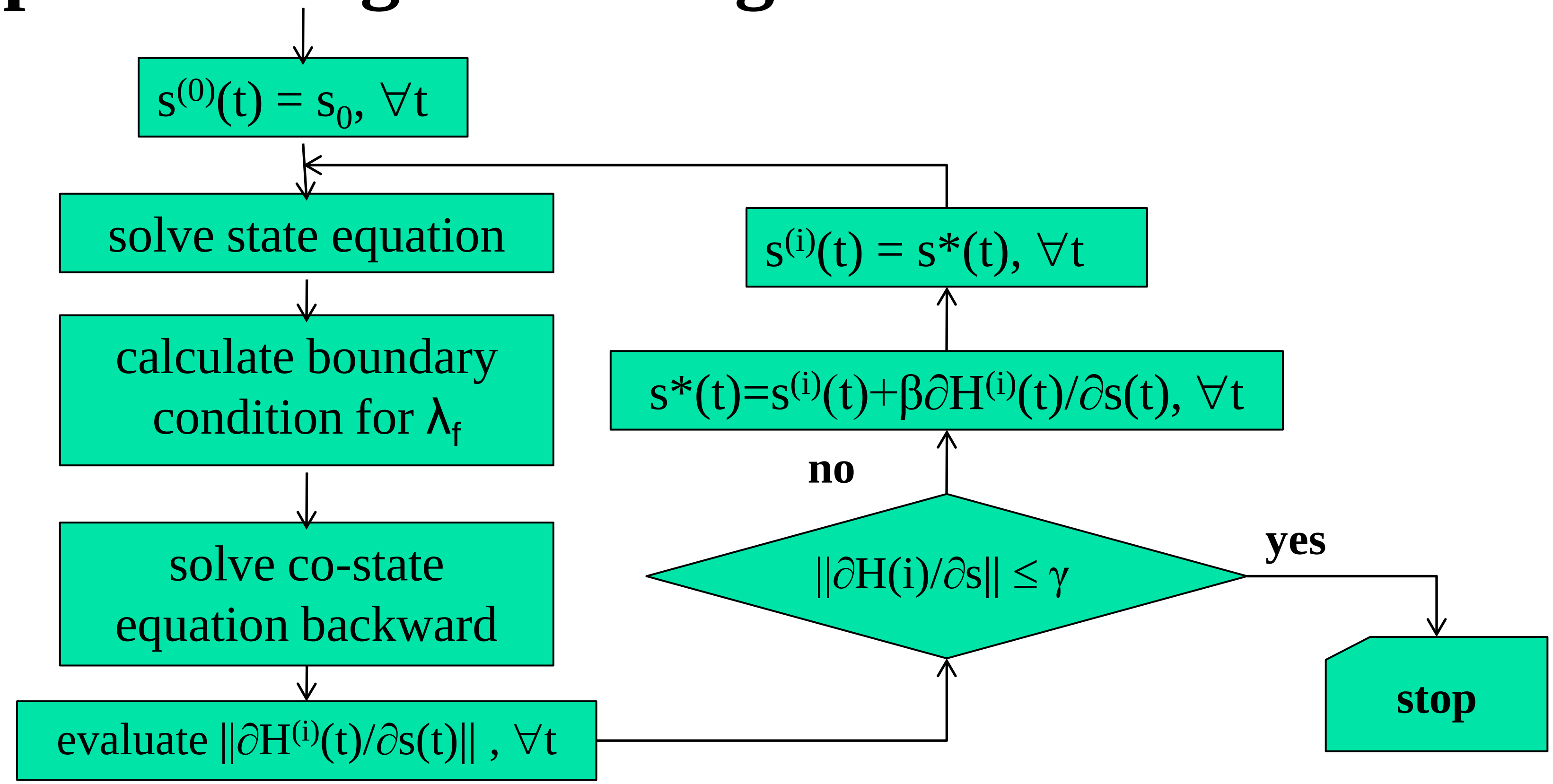
$$\begin{cases} \dfrac{dg^*(t)}{dt} = r_g g^* \left(1 - s^* - c_{gg} g^* - c_{wg} w^*\right) \\ \dfrac{dw^*(t)}{dt} = r_w \left[\alpha + w^* \left(1 - c_{gw} g^* - c_{ww} w^*\right)\right] \end{cases} \qquad X(t_0) = \left(g_0, w_0\right)^T$$

$$\begin{cases} \dot{\lambda}_1^*(t) = -r_g \lambda_1^* \left(1 - s^* - 2c_{gg} g^* - c_{wg} w^*\right) + r_w c_{gw} w^* \lambda_2^* \\ \dot{\lambda}_2^*(t) = r_g c_{wg} g^* \lambda_1^* - r_w \left[1 - c_{gw} g^* - 2c_{ww} w^*\right] \lambda_2^* \end{cases} \qquad \begin{cases} \lambda_1^*(t_f) = -h_1\left(g^*(t_f) - g^e(t_f)\right) \\ \lambda_2^*(t_f) = -h_2\left(w^*(t_f) - w^e(t_f)\right) \end{cases}$$

$$s^*(t) = \frac{r_g}{r} g^*(t) \lambda_1^*(t)$$

Model and Algorithm

Solution of two-point boundary-value problem - gradient algorithm



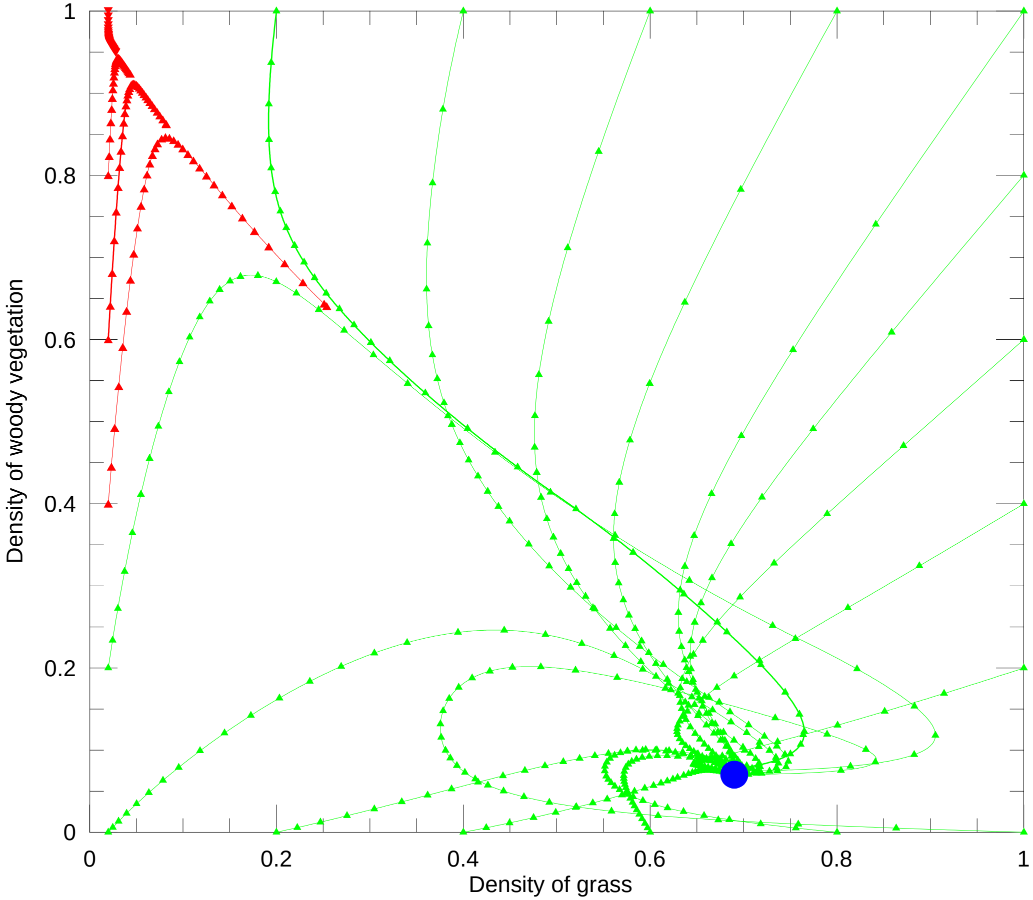
Numerical Experiment

Conditions

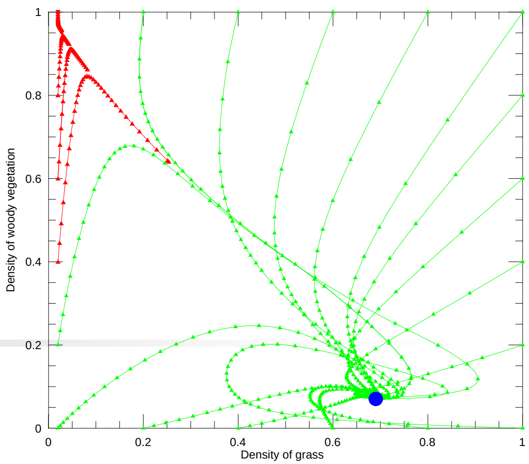
- Planning period = 10 years
- Desired final state $(g^e(t_f), w^e(t_f)) = (0.69, 0.07)$
- $r = 1$, $hg = hw = 10000$
- $s^{(0)}(t) = 0.486$
- Initial condition: from four sides in phase diagram

Numerical Experiment

Phase diagram



Numerical Experiment



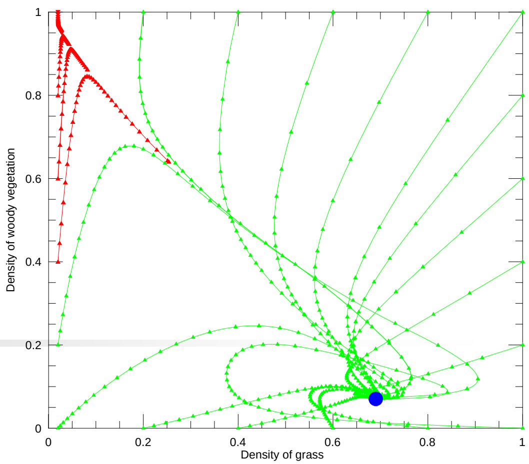
Observations

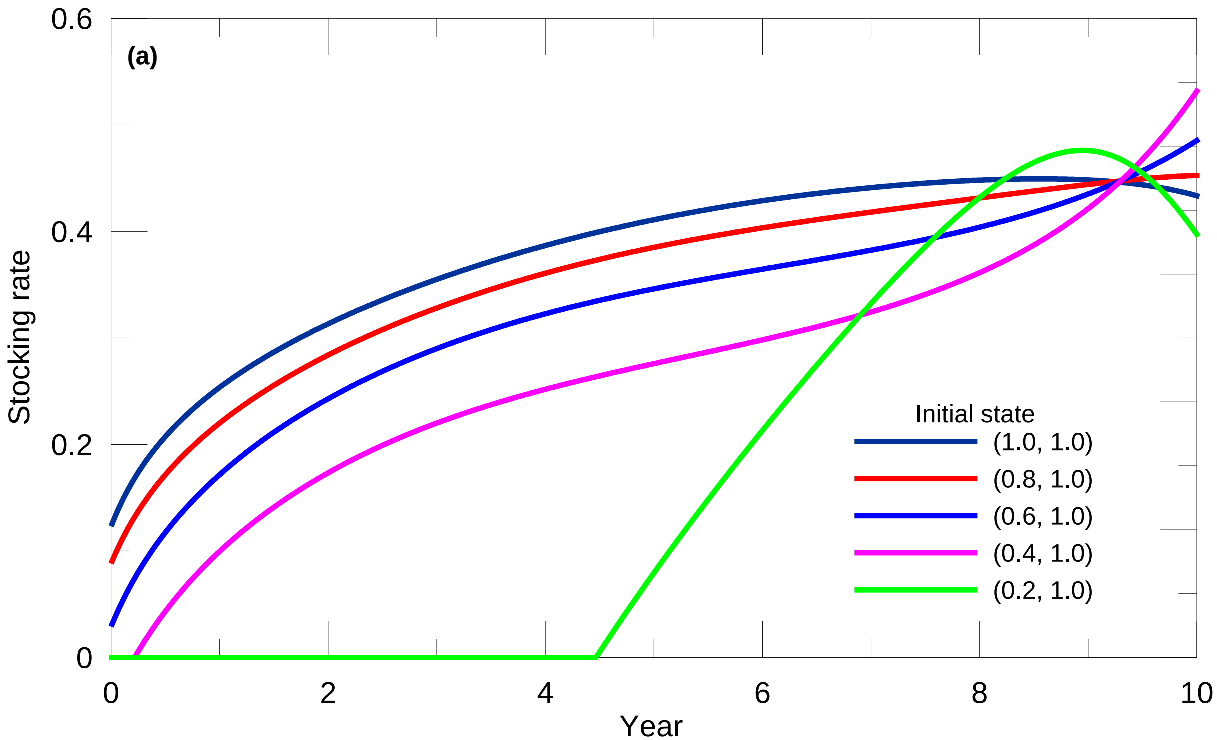
- The system can transfer to the specified desired final state from most of initial states.

- The system still move toward the desired final state through stopping raising grazing cattle in the rangeland in the situation woody vegetation dominates grass. Once the system falls in this situation, it will need to take a long time for recovering the system resilience.

Numerical Experiment

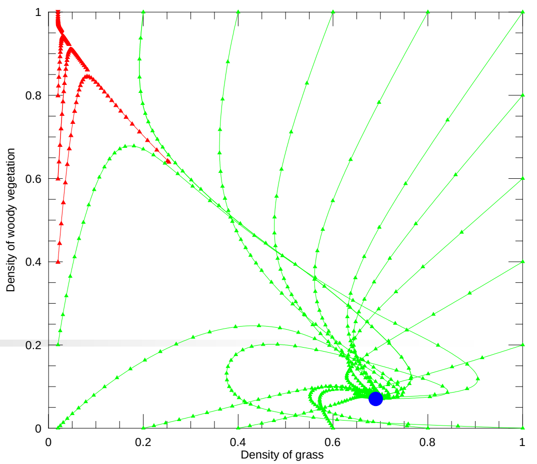
Optimal stocking rates

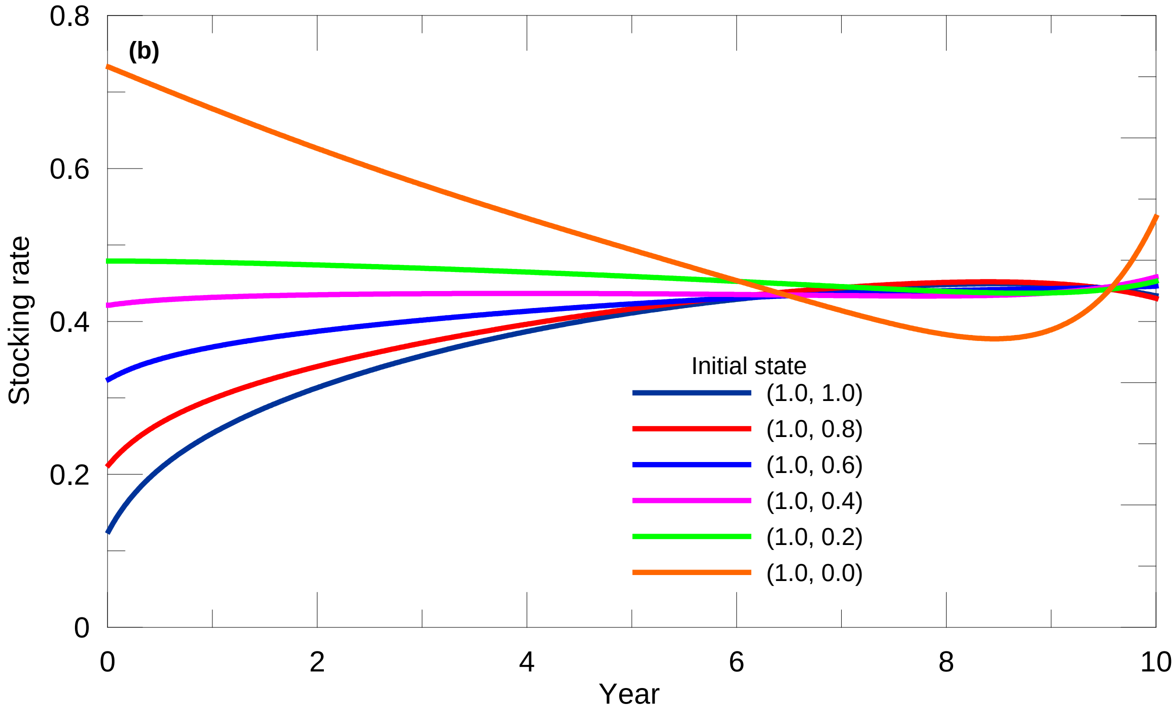




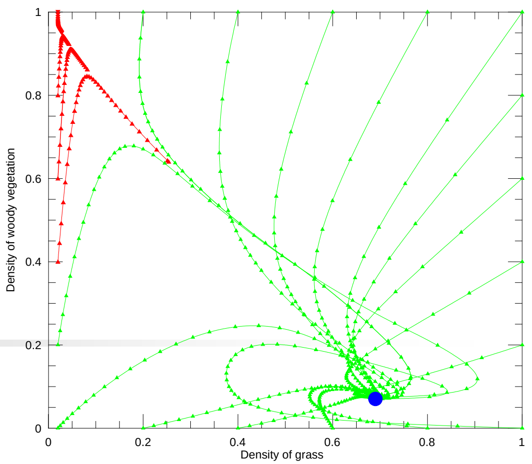
Numerical Experiment

Optimal stocking rates (cont)

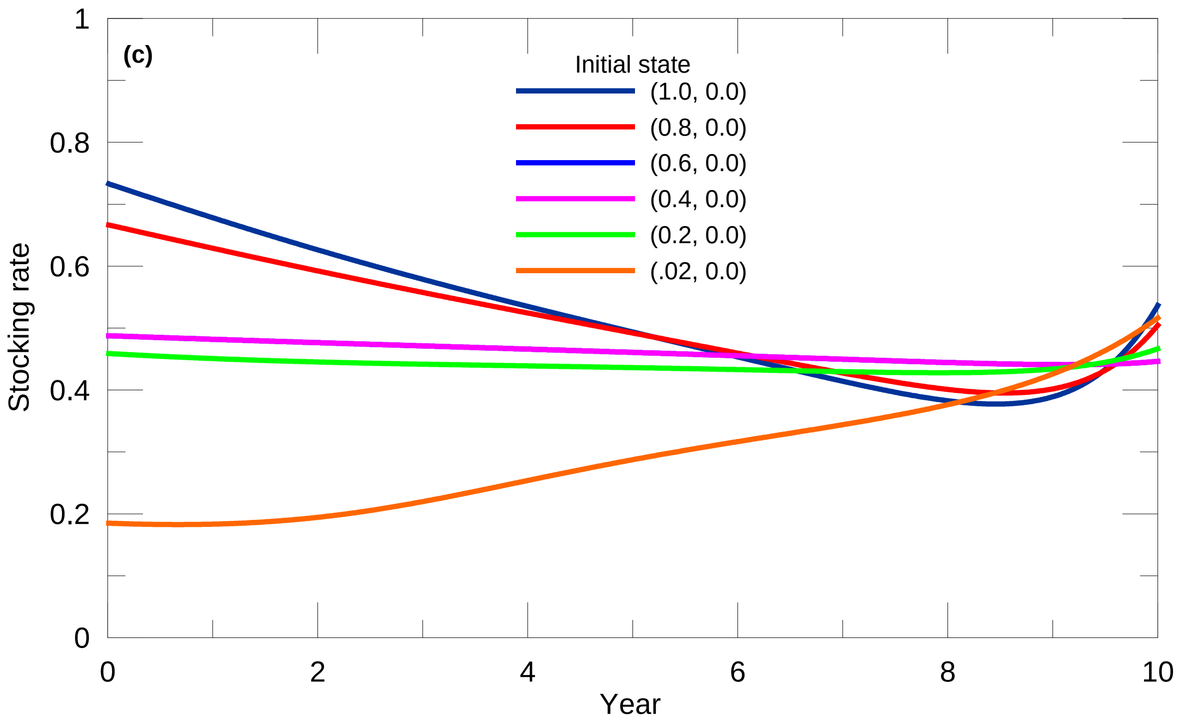




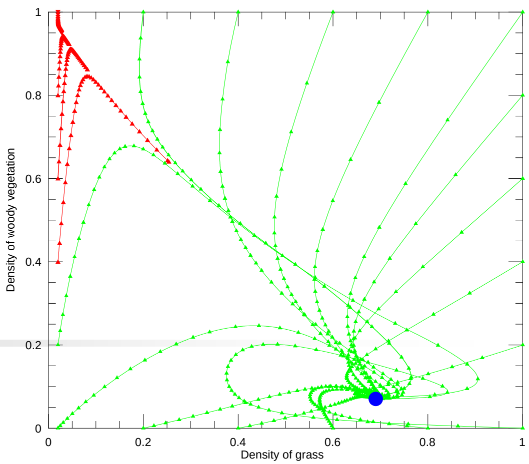
Numerical Experiment



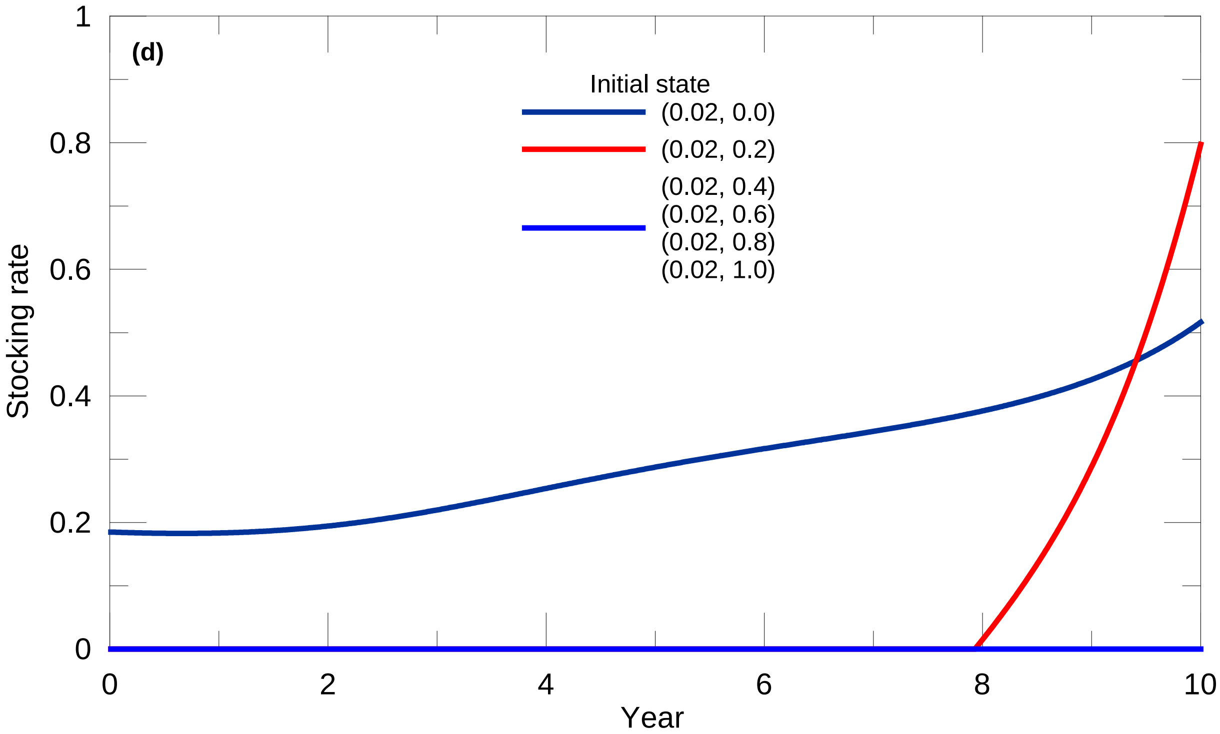
Optimal stocking rates (cont)



Numerical Experiment

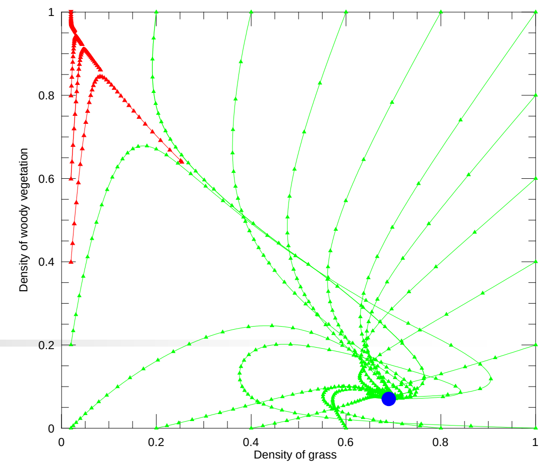


Optimal stocking rates (cont)



$\bar{s}$

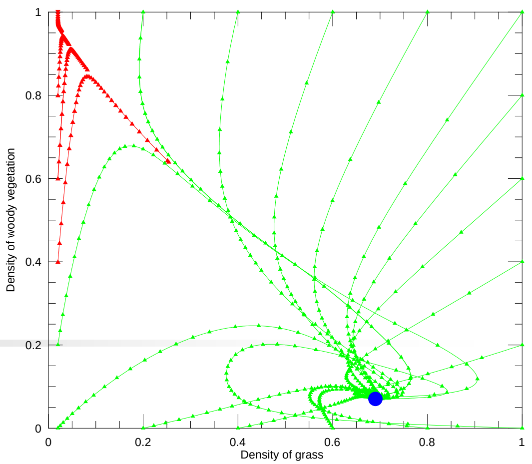
Numerical Experiment



Annual average stocking rates

(g_0, w_0)	s	(g_0, w_0)	s	(g_0, w_0)	s	(g_0, w_0)	s
(1.0, 1.0)	0.379	(1.0, 0.8)	0.394	(0.8, 0.0)	0.505	(.02, 0.2)	0.066
(0.8, 1.0)	0.357	(1.0, 0.6)	0.413	(0.6, 0.0)	0.488	(.02, 0.4)	0.000
(0.6, 1.0)	0.323	(1.0, 0.4)	0.435	(0.4, 0.0)	0.461	(.02, 0.6)	0.000
(0.4, 1.0)	0.268	(1.0, 0.2)	0.458	(0.2, 0.0)	0.439	(.02, 0.8)	0.000
(0.2, 1.0)	0.174	(1.0, 0.0)	0.516	(.02, 0.0)	0.294	(.02, 1.0)	0.000

Numerical Experiment



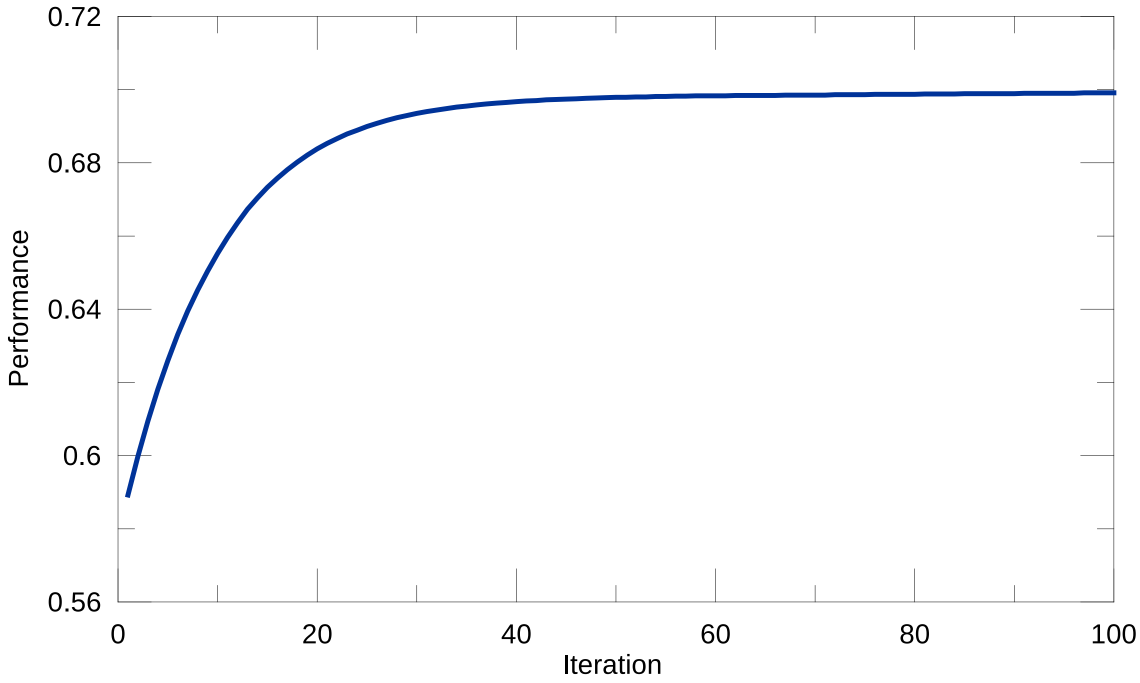
Observations

- The optimal strategies can obtain maximum economic profit from raising grazing cattle.
- The optimal strategies can significantly improve the resilience of the rangeland so that the system operate in a stable region.
- Where the initial state is located directly affects operation of the system and economic profit.

Discussion

Algorithm convergence

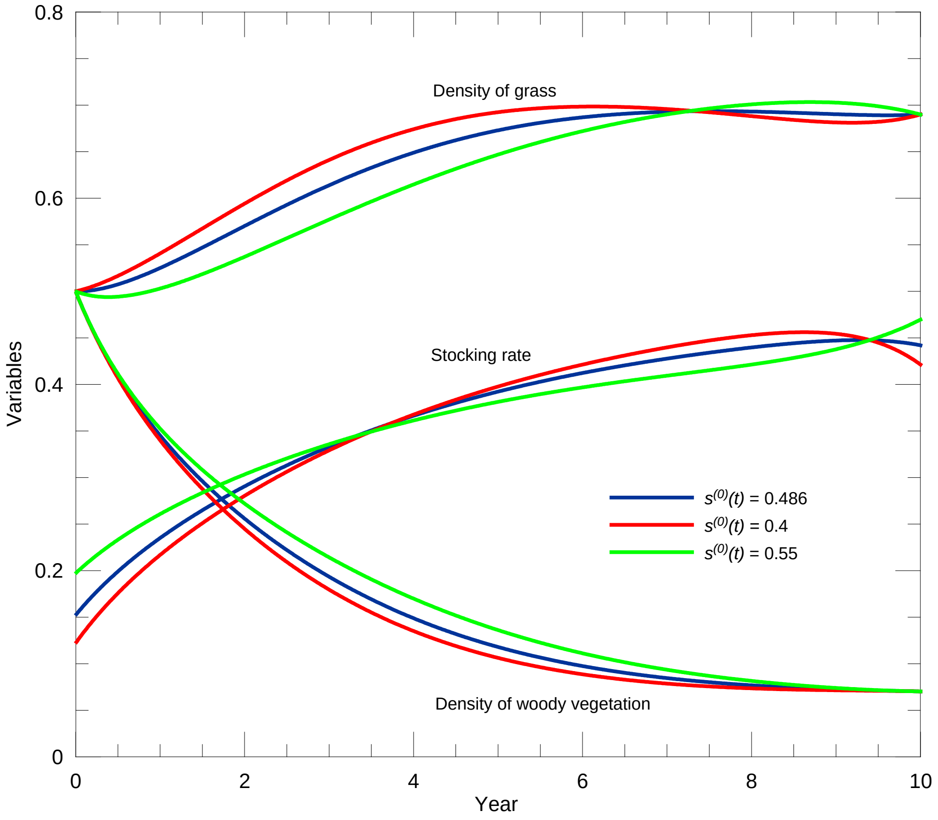
- $(g_0, w_0) = (0.5, 0.5)$, desired final state (0.69, 0.07)
- $s^{(0)}(t) = 0.486$



Discussion

Impact of initial stocking rate

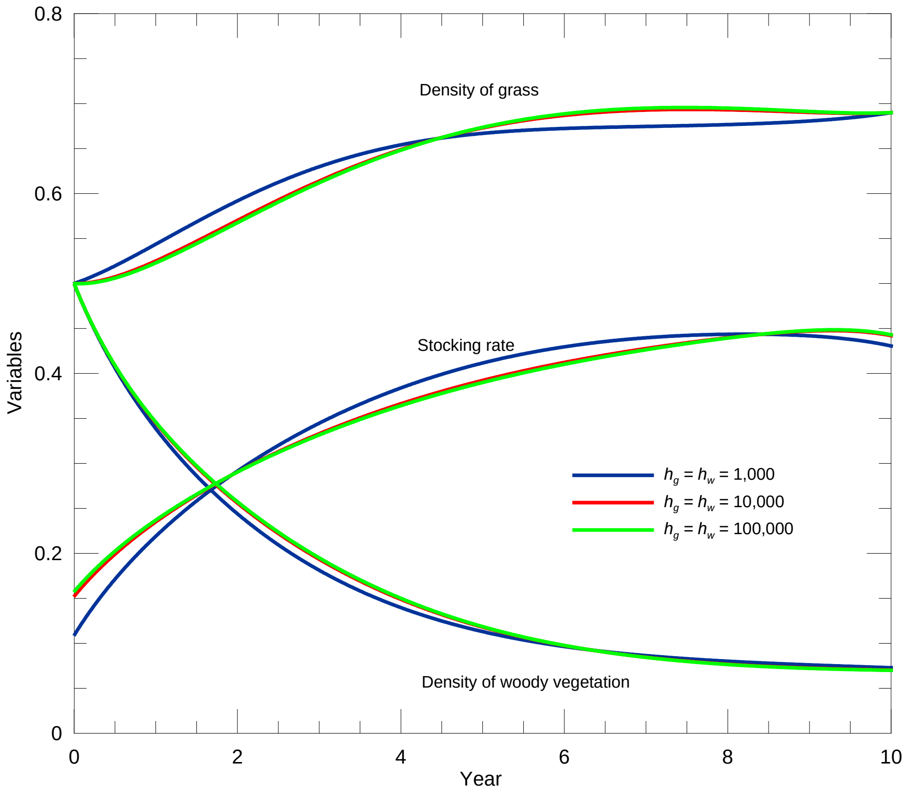
- $s^{(0)}(t)$ = 0.486, 0.4, 0.55



Discussion

Impact of penalty coefficients *hg* and *hw*

- *hg* =*hw* = 1,000, 10,000, 100,000



Conclusions

- The initial state of the rangeland has an important impact on optimal strategies. When grass is dominant, the system can obtain maximum economic profit without loss of resilience. If the rangeland is dominated by woody vegetation, the system can still move toward the desired final state through controlling stocking rate so that it can restore resilience and re-obtain development sustainability. The optimal strategies can significantly improve the resilience of the rangeland so that the system operate in a stable region.

- If the rangeland is almost covered by woody vegetation with a lower density of grass such as $s <= 0.02$, a best strategy is to stop raising grazing cattle until the system recovers resilience. If we would be still eager for instant success and quick profits in this situation, the system would inevitably become worse until it loses all resilience. Once the system falls into this situation, the rangeland will become a “dead” land for grazing cattle and will difficultly restore its resilience again.

Conclusions

- The desired final state in planning period reflects the requirement for development sustainability of the rangeland in long-term. It is not only a terminal point of this planning period, but also is the initial state for next planning period. In other words, it is a junction between short-term and long-term profits. It is always a good choice to use the second stable equilibrium state as the desired final state.

- Optimal algorithm can be efficiently applied to obtain optimal strategies through selection of initial stocking rate and penalty coefficients. It is a good guidance to take 0.486 as initial stocking rate while selection of penalty coefficients should have the penalty terms comparable with original objective function.

- Sustainability, resilience, economic profit are consistent

Thank You!

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