Global Temperature Change and Sea-Level Rise

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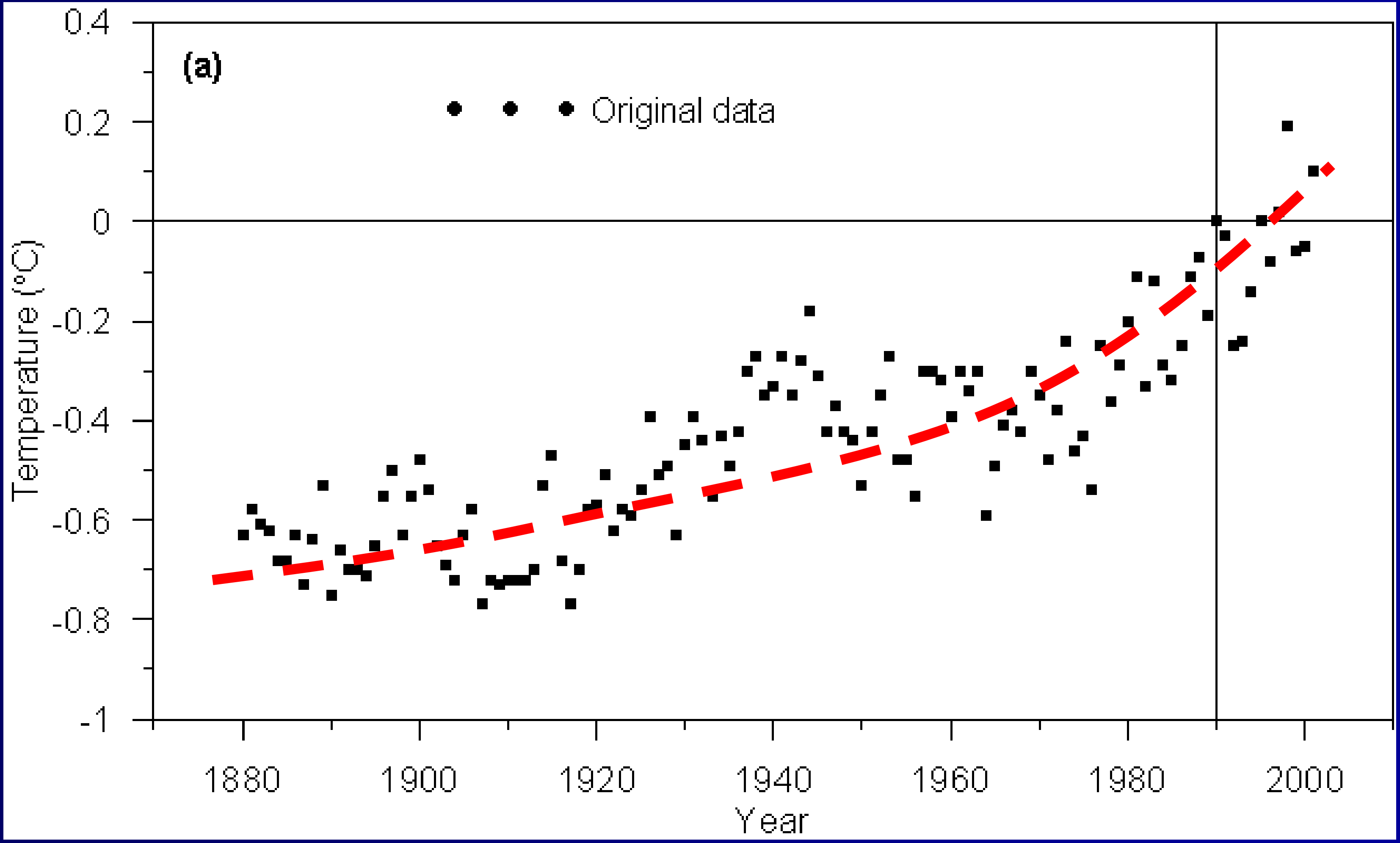
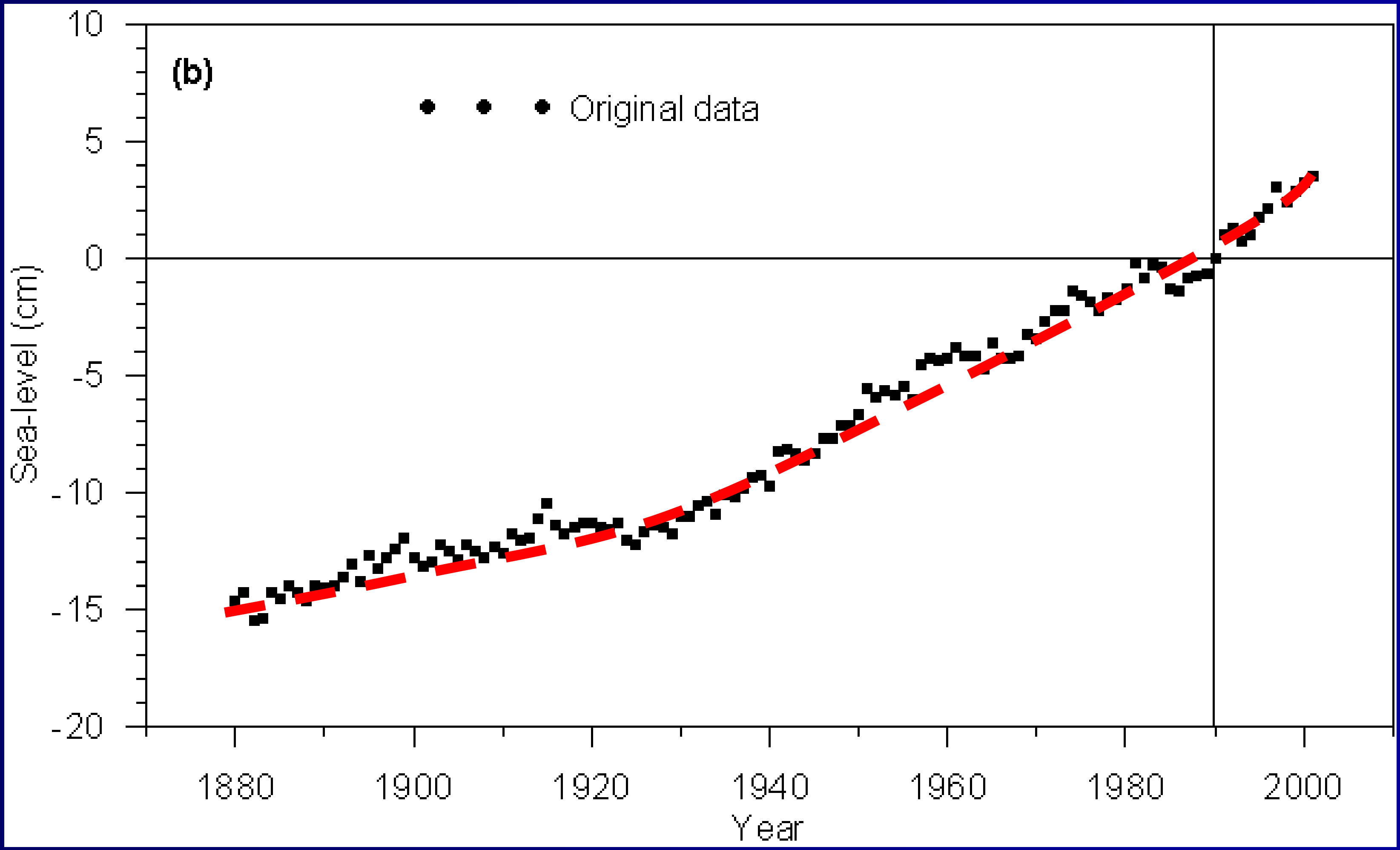
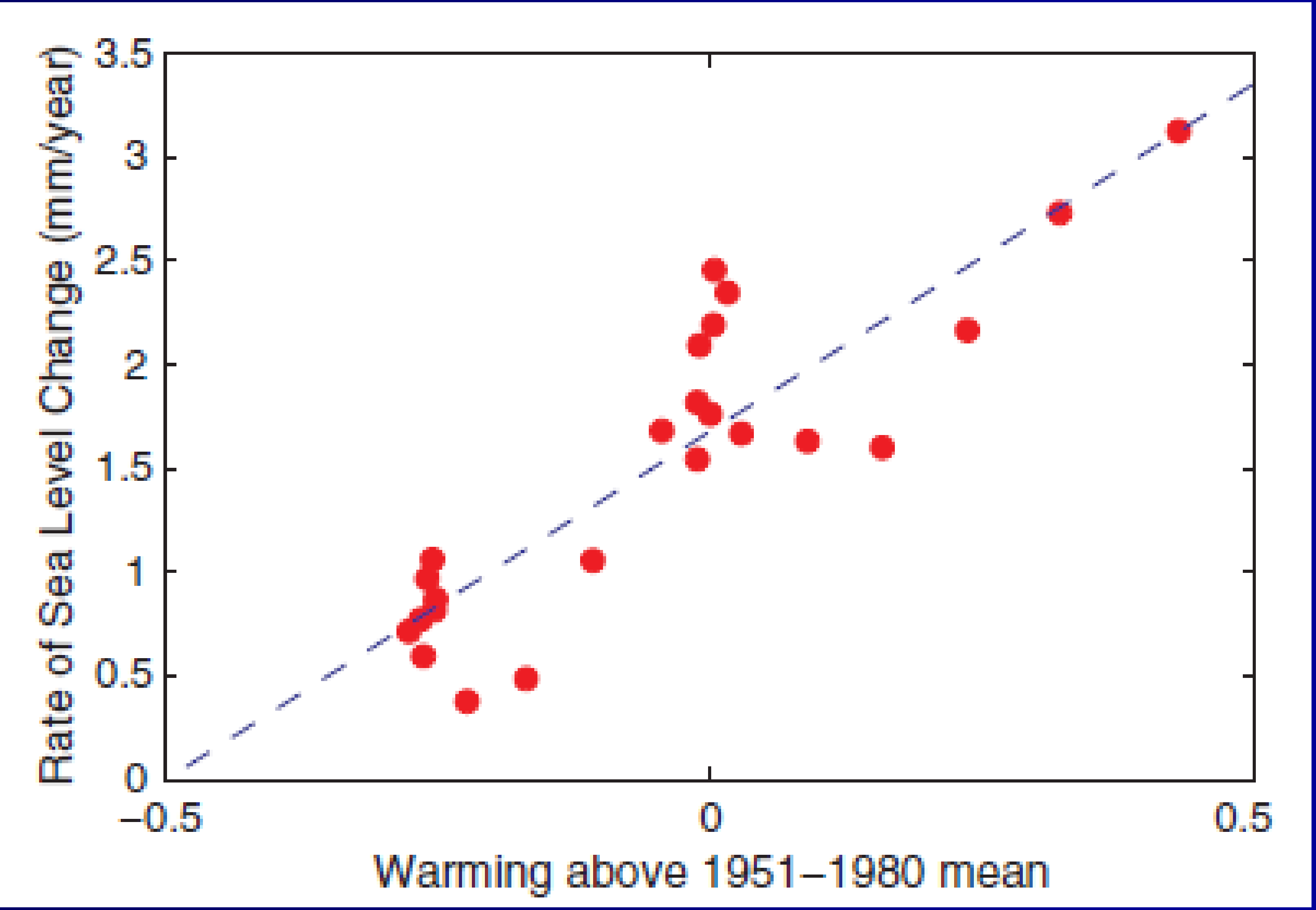
Global Temperature Change:



Global Sea-Level Rise:



Correlation:

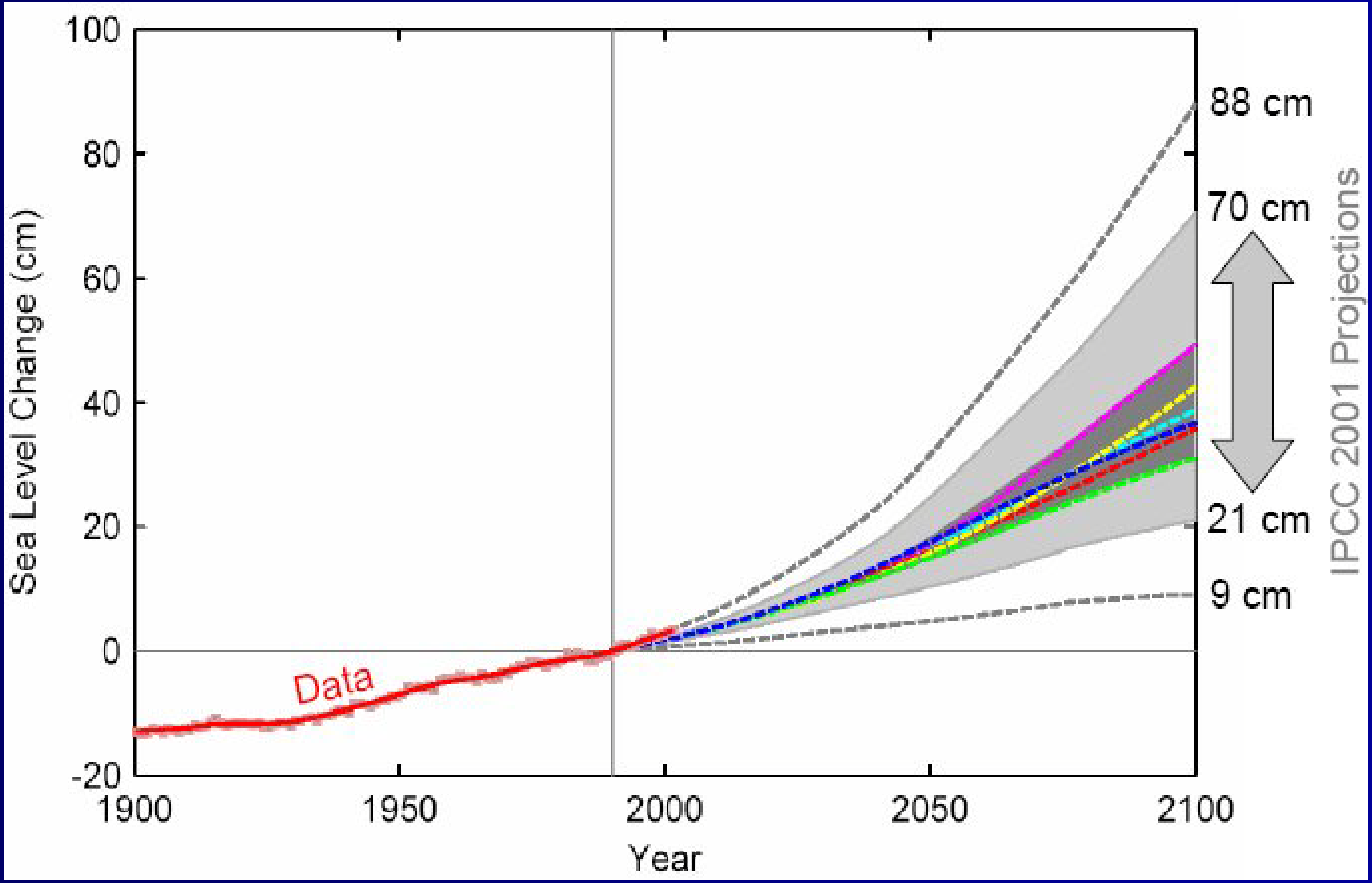


What does IPCC say?

The Summary for Policy Makers (SPM) released recently provides the following table of sea level rise projections:

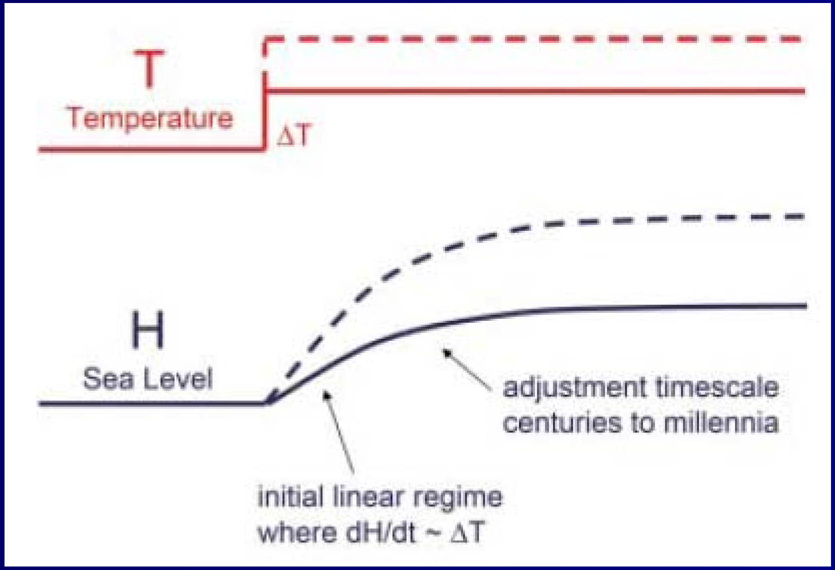
Case	Sea Level Rise (m at 2090-2099 relative to 1980-1999) Model-based range excluding future rapid dynamical changes in ice flow
B1 scenario	0.18 – 0.38
A1T scenario	0.20 – 0.45
B2 scenario	0.20 – 0.43
A1B scenario	0.21 – 0.48
A2 scenario	0.23 – 0.51
A1FI scenario	0.26 – 0.59

What does IPCC say?



Semi-empirical models:

Rahmstorf's Study (Science, Vol. 315 pp.19, 2007)

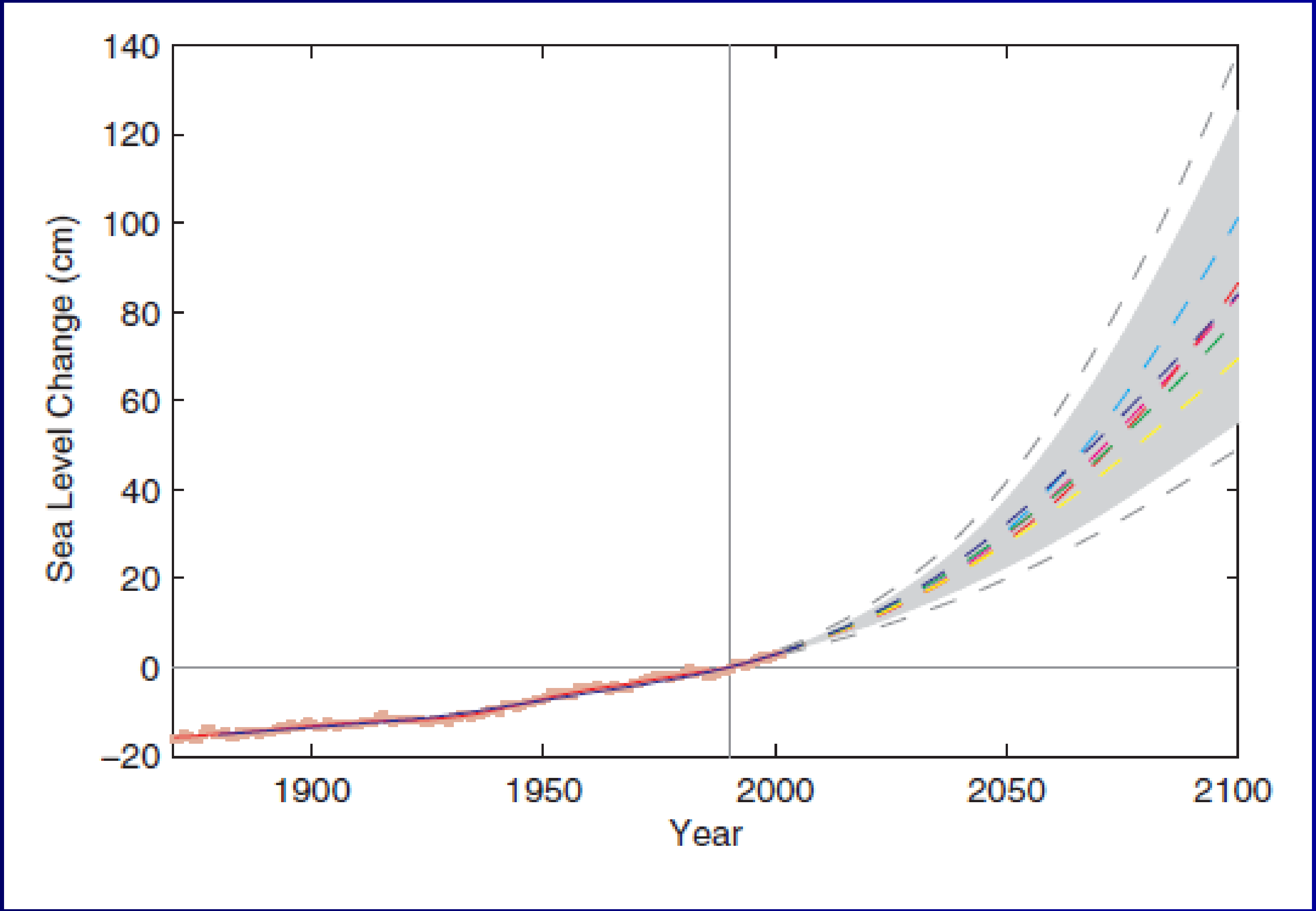


$$\frac{\partial H}{\partial t} = a\left(T - T_o\right)$$

$$H\left(t\right) = a\int_{t_o}^{t}\left(T\left(\tau\right) - T_o\right)d\tau$$

Semi-empirical models:

Rahmstorf's Study (Science, Vol. 315 pp.19, 2007)



Semi-empirical models:

Rahmstorf's Study (Science, Vol. 315 pp.19, 2007)

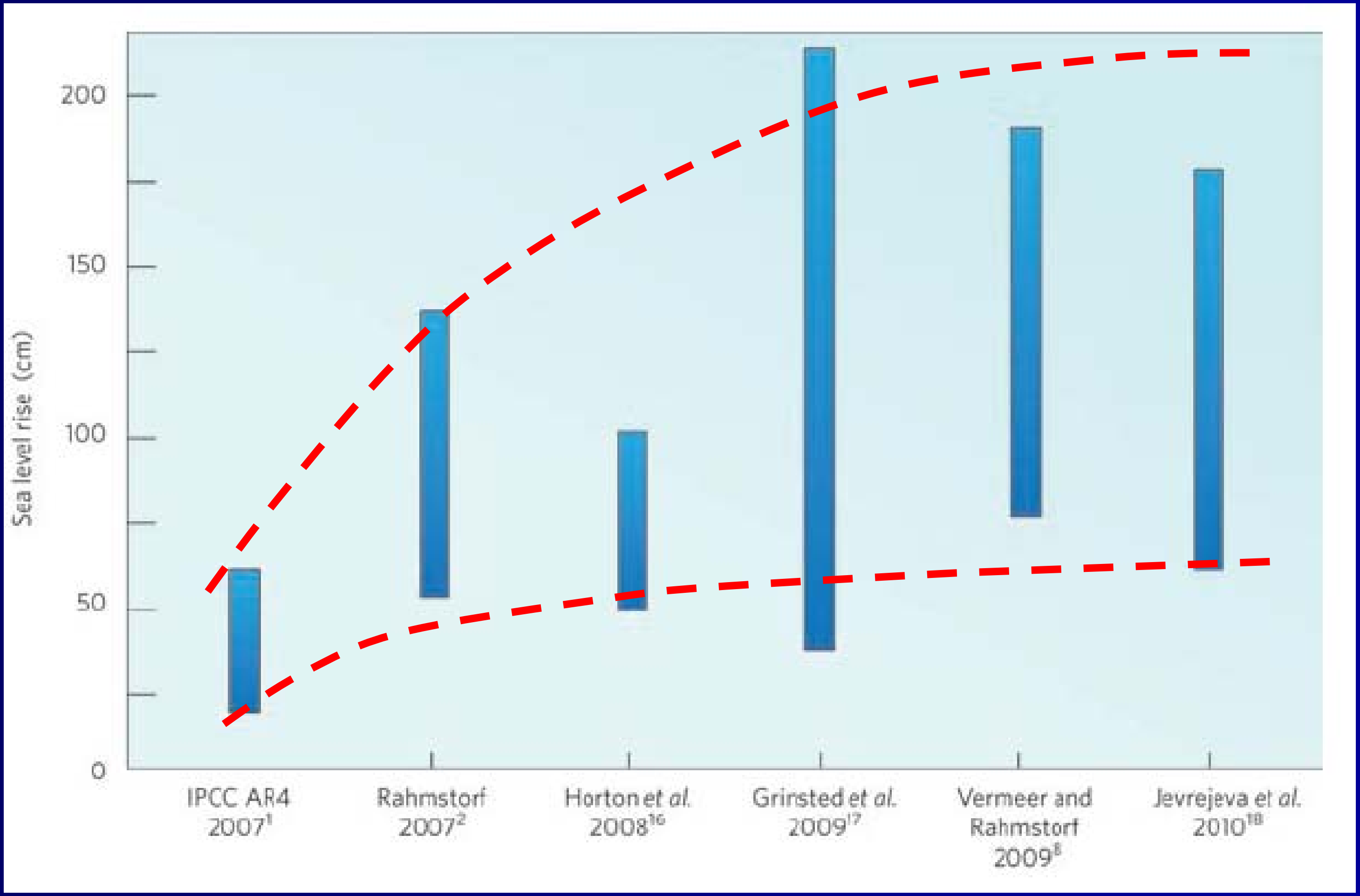
Sea-Level Rise Interval Predicted:

$\{0.5-1.4m\}$ above 1990 level

IPCC Interval Predicted:

$\{0.09-0.88m\}$ above 1990 level

IPCC and other Models:



Everything should be made as simple as possible, but not simpler.

A. Einstein

A Dynamic Systems Model

Expected Relationship:

- Earlier studies showed that the relationship between T & H is linear.

Hypothesis:

- Temperature: $T = f_1\left(T, H, U, c_1\right)$
- Sea-Level: $H = f_2\left(T, H, U, c_2\right)$

Proposed Model:

$$\frac{dT(t)}{dt} = a_{11}T(t) + a_{12}H(t) + a_{13}U_i\left(t\right) + c_1$$

$$\frac{dH(t)}{dt} = a_{21}T(t) + a_{22}H(t) + a_{23}U_i\left(t\right) + c_2$$

Proposed Model: (Simplified)

$$\frac{dT(t)}{dt} = a_{11}T(t) + a_{12}H(t) + c_1$$

$$\frac{dH(t)}{dt} = a_{21}T(t) + a_{22}H(t) + c_2$$

Proposed Model:

$$\mathbf{X}(t) = \left(T(t),\ H(t)\right)^{\tau} = \left(x_1(t),\ x_2(t)\right)^{\tau}$$

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad \mathbf{C} = \begin{Bmatrix} c_1 \\ c_2 \end{Bmatrix}$$

$$\frac{d\mathbf{X}}{dt} = \mathbf{AX}(t) + \mathbf{C}$$

Proposed Discrete Model:

$$\mathbf{X}(k+1) = \boldsymbol{\Phi}\mathbf{X} \quad \boldsymbol{\Omega} \ (k) +$$

$$\boldsymbol{\Phi} = \left(\mathbf{I} + \mathbf{A}\Delta t\right); \quad \boldsymbol{\Omega} = \mathbf{C}\Delta t$$

LSM Model to determine a_{ij} & c_i :

$$F^{*} = \underset{\varphi_i}{\text{minimize}} \left\{ \left(\mathbf{Y}_i \boldsymbol{\Lambda} - \boldsymbol{\varphi}_i \right)^{\tau} \mathbf{Y} \left({}_i \boldsymbol{\Lambda} - \boldsymbol{\varphi}_i \right) \right\}$$

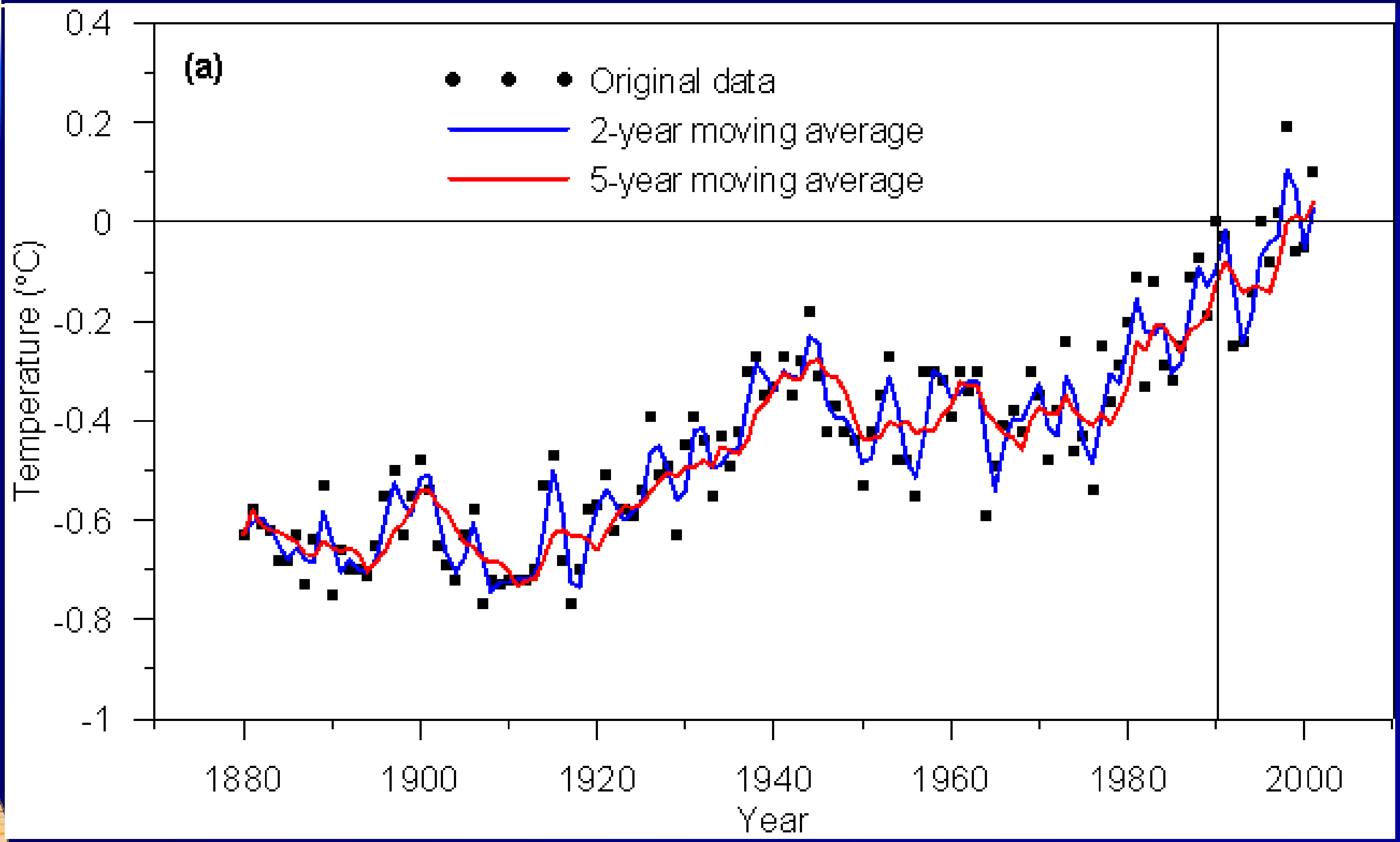
Confidence Interval:

$$\left.\begin{aligned}
\hat{T}_{CI}(k) &= \hat{T}(k) \pm t_{\alpha/2,n-4}\sqrt{\hat{\sigma}_T^2\left(1+\hat{Z}(k)^{\tau}\left(\mathbf{\Lambda}^{\tau}\mathbf{\Lambda}\right)^{-1}\hat{Z}(k)\right)} \\
\hat{H}_{CI}(k) &= \hat{H}(k) \pm t_{\alpha/2,n-4}\sqrt{\hat{\sigma}_H^2\left(1+\hat{Z}(k)^{\tau}\left(\mathbf{\Lambda}^{\tau}\mathbf{\Lambda}\right)^{-1}\hat{Z}(k)\right)}
\end{aligned}\right\}$$

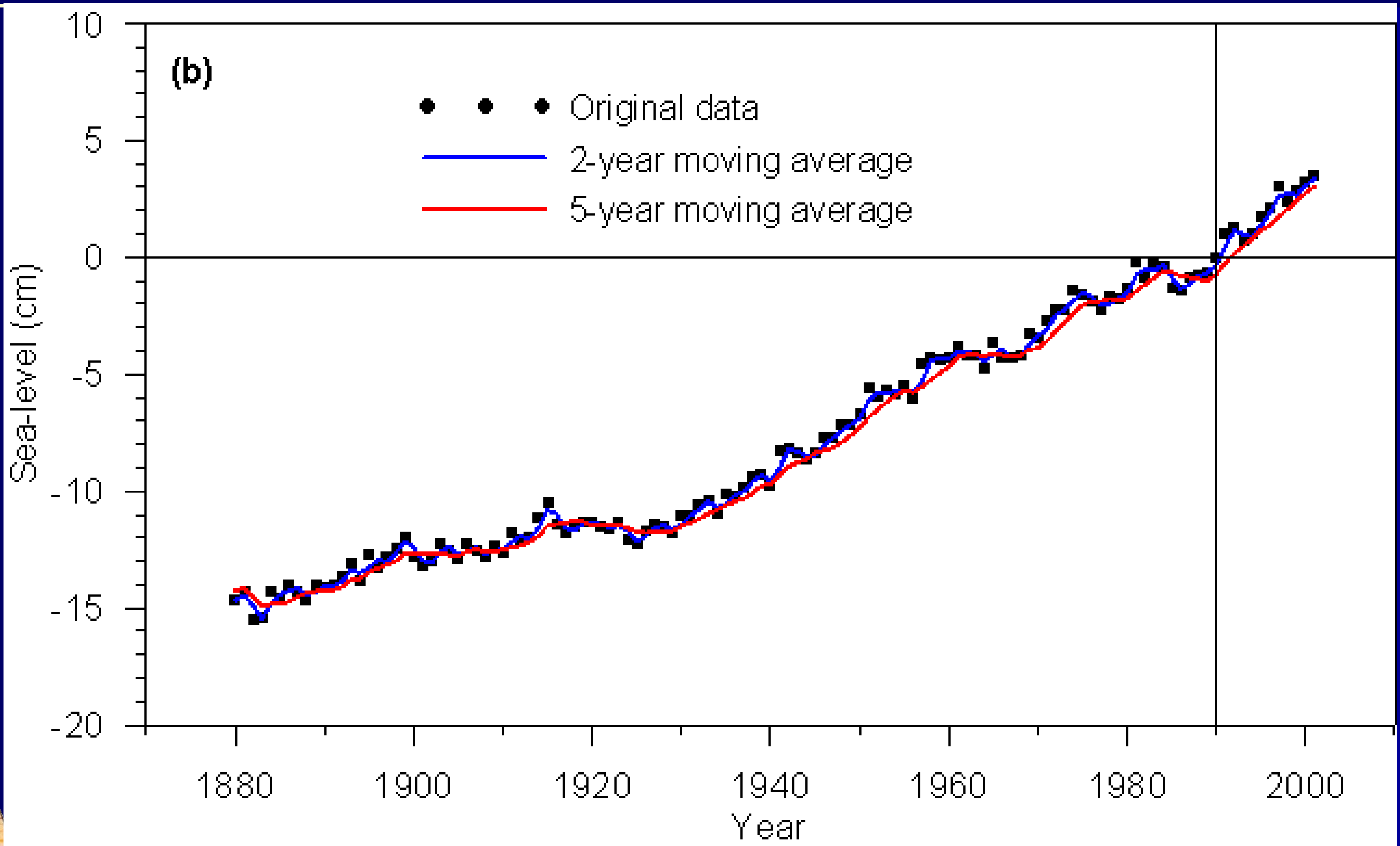
Application:

- 2-year moving average outcome is used for both state variables.

Temperature Data:



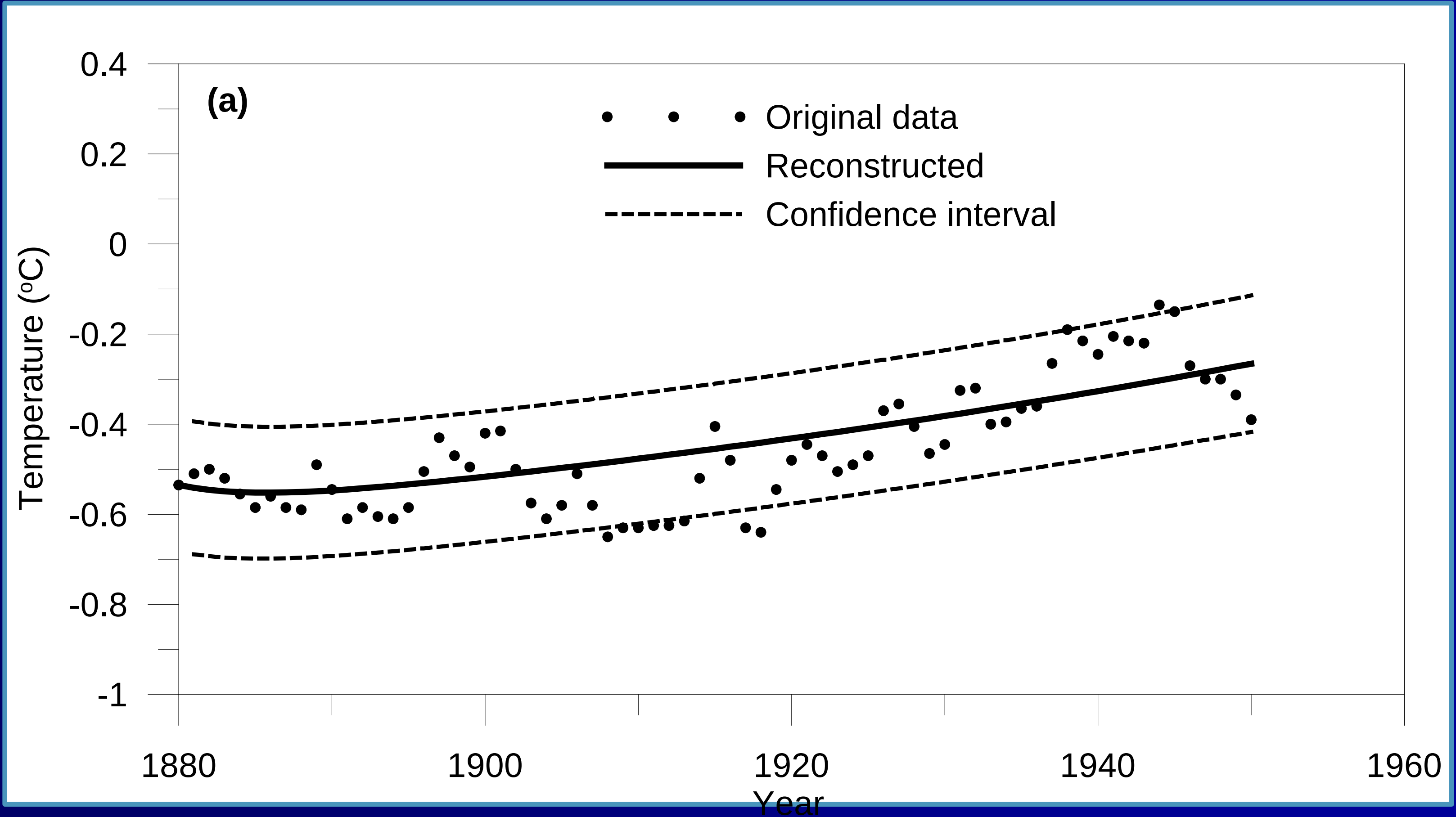
Sea-Level Data:



Matrix Coefficients:

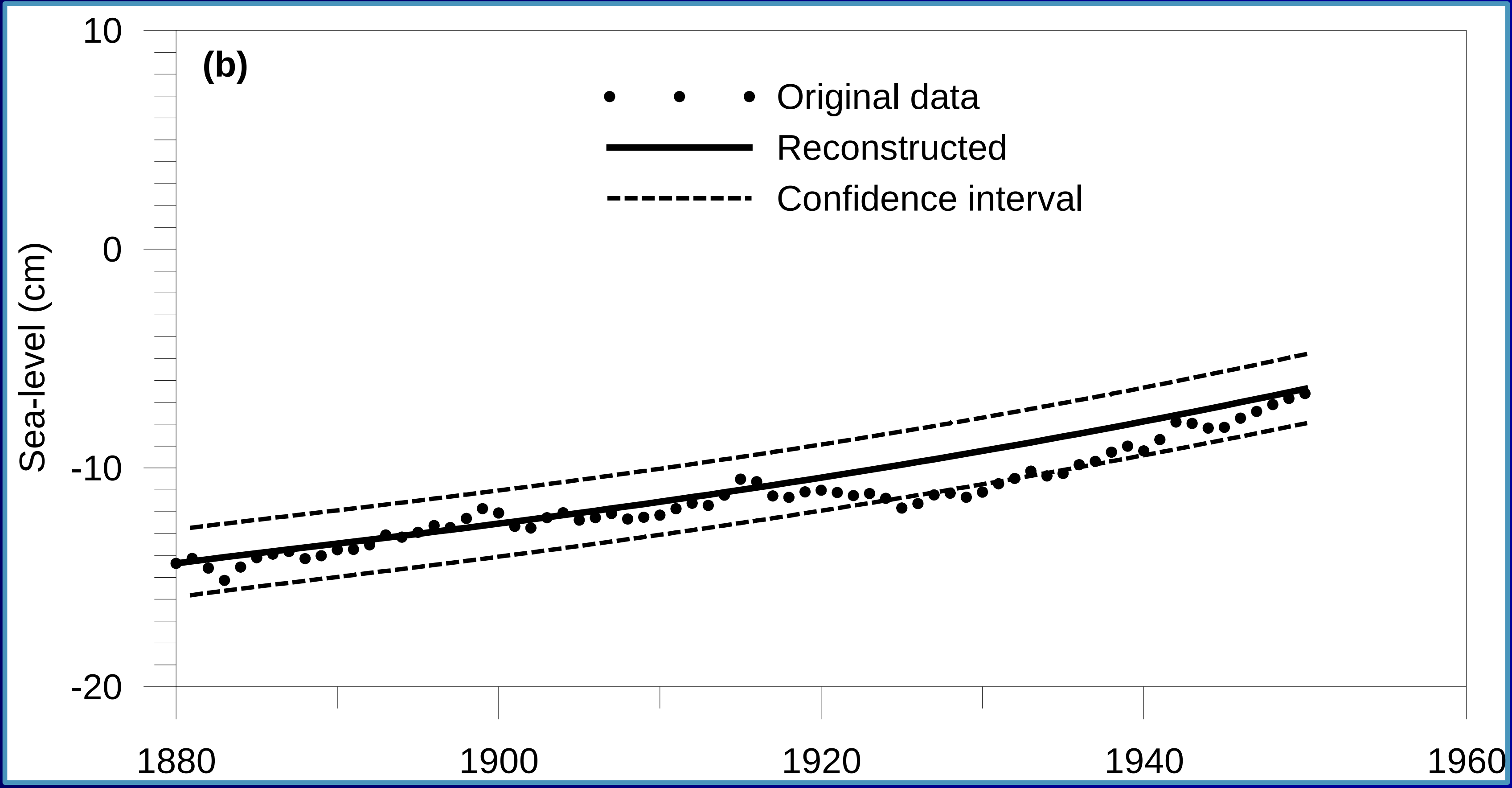
Data used	Discrete system (Φ, Ω)	Continuous system (A, C)
1880 - 1950	$\begin{bmatrix} 0.8337 & 0.0072 \\ 0.3441 & 0.9960 \end{bmatrix} \begin{Bmatrix} 0.0083 \\ 0.2234 \end{Bmatrix}$	$\begin{bmatrix} -0.1663 & 0.0072 \\ 0.3441 & -0.0040 \end{bmatrix} \begin{Bmatrix} 0.0083 \\ 0.2234 \end{Bmatrix}$
1880 - 2001	$\begin{bmatrix} 0.8074 & 0.0068 \\ 0.4115 & 0.9956 \end{bmatrix} \begin{Bmatrix} -0.0110 \\ 0.2585 \end{Bmatrix}$	$\begin{bmatrix} -0.1926 & 0.0068 \\ 0.4115 & -0.0045 \end{bmatrix} \begin{Bmatrix} -0.0110 \\ 0.2585 \end{Bmatrix}$

Temperature (1880-1950): (90%) Conf. Int.



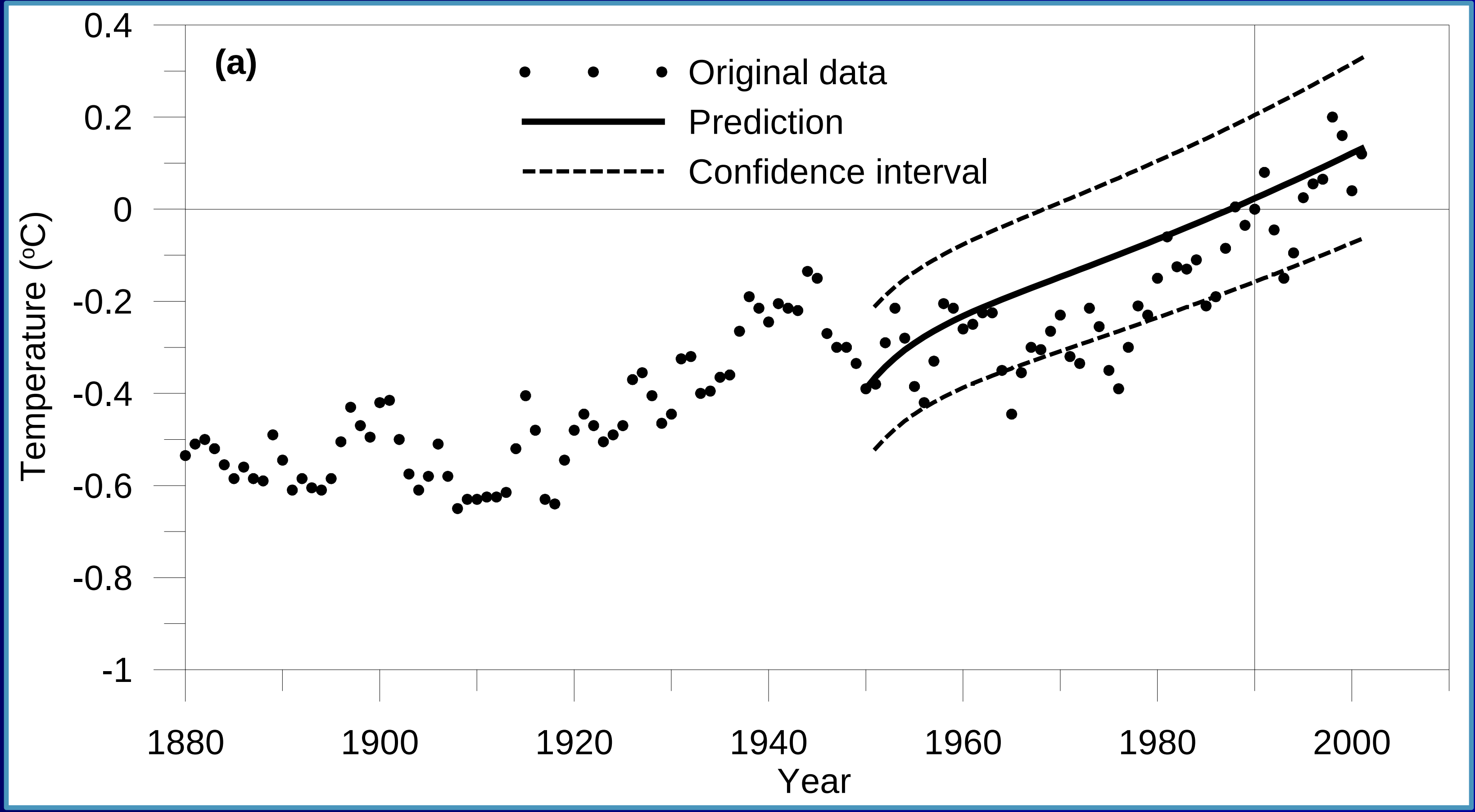
$R^2 = 0.6$

Sea-Level (1880-1950): (90%) Conf. Int.



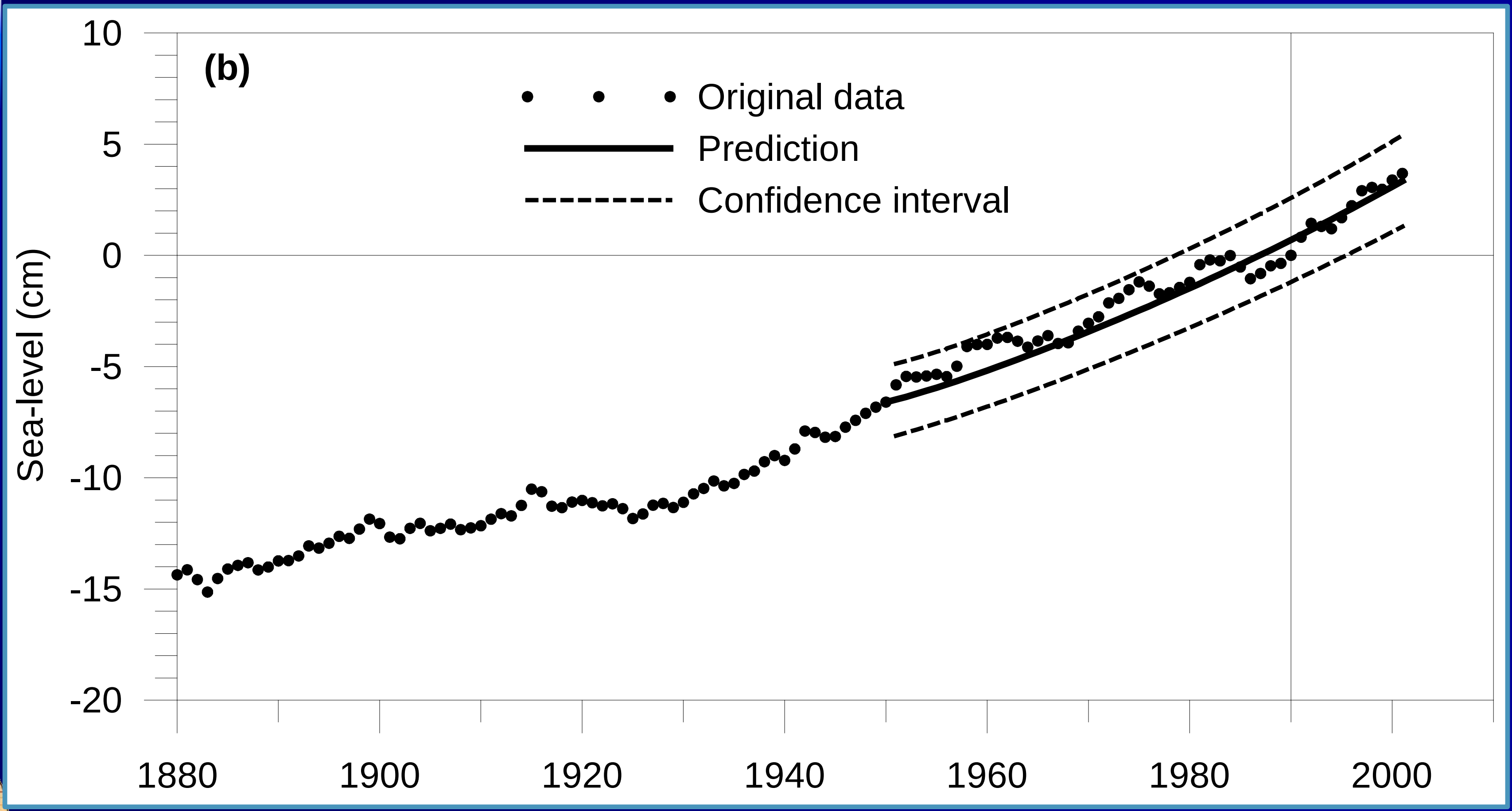
$R^2 = 0.8$

Temperature (1950-2001): (90%) Conf. Int.



$R^2 = 0.5$

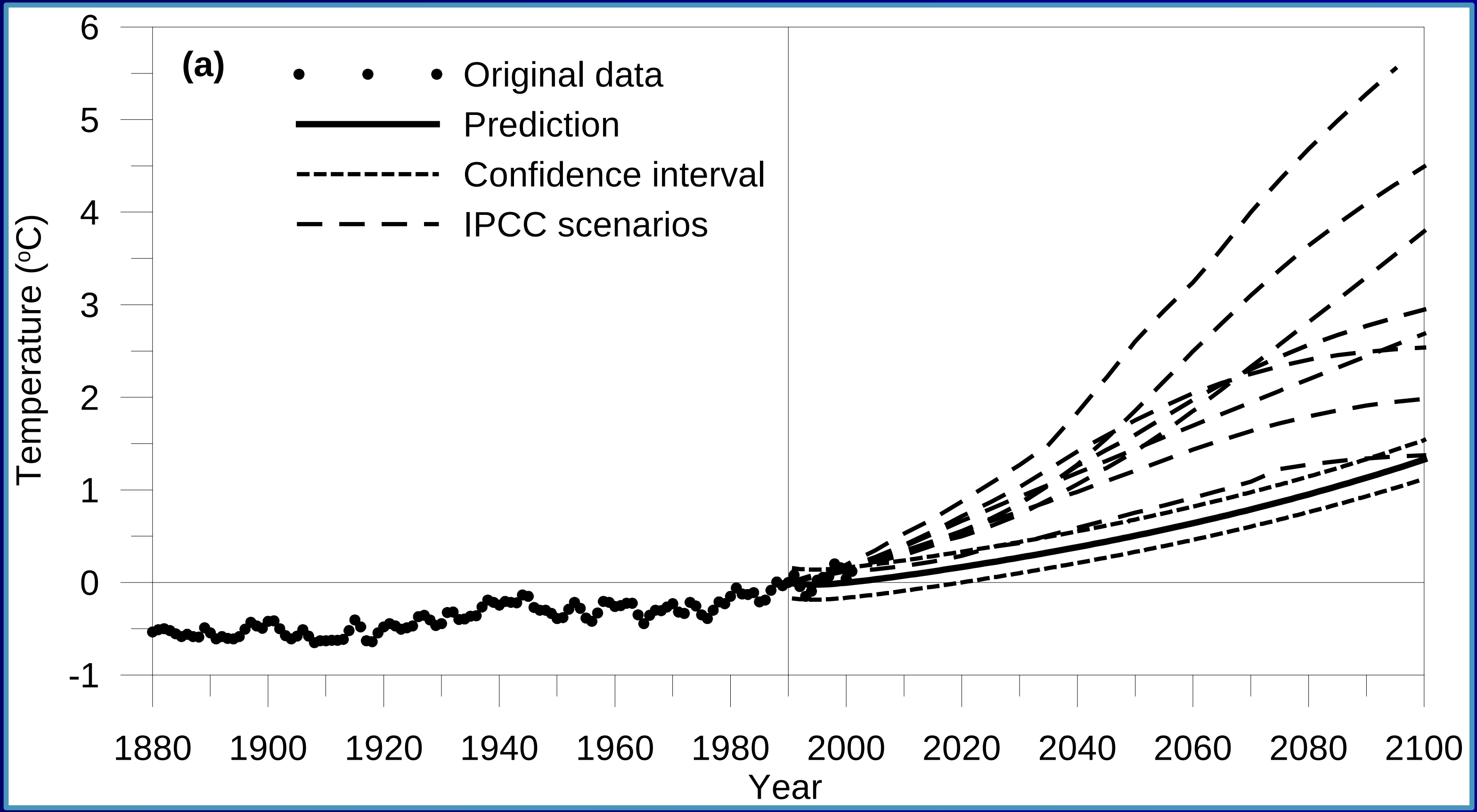
Sea-Level (1950-2001): (90%) Conf. Int.



$R^2 = 0.9$

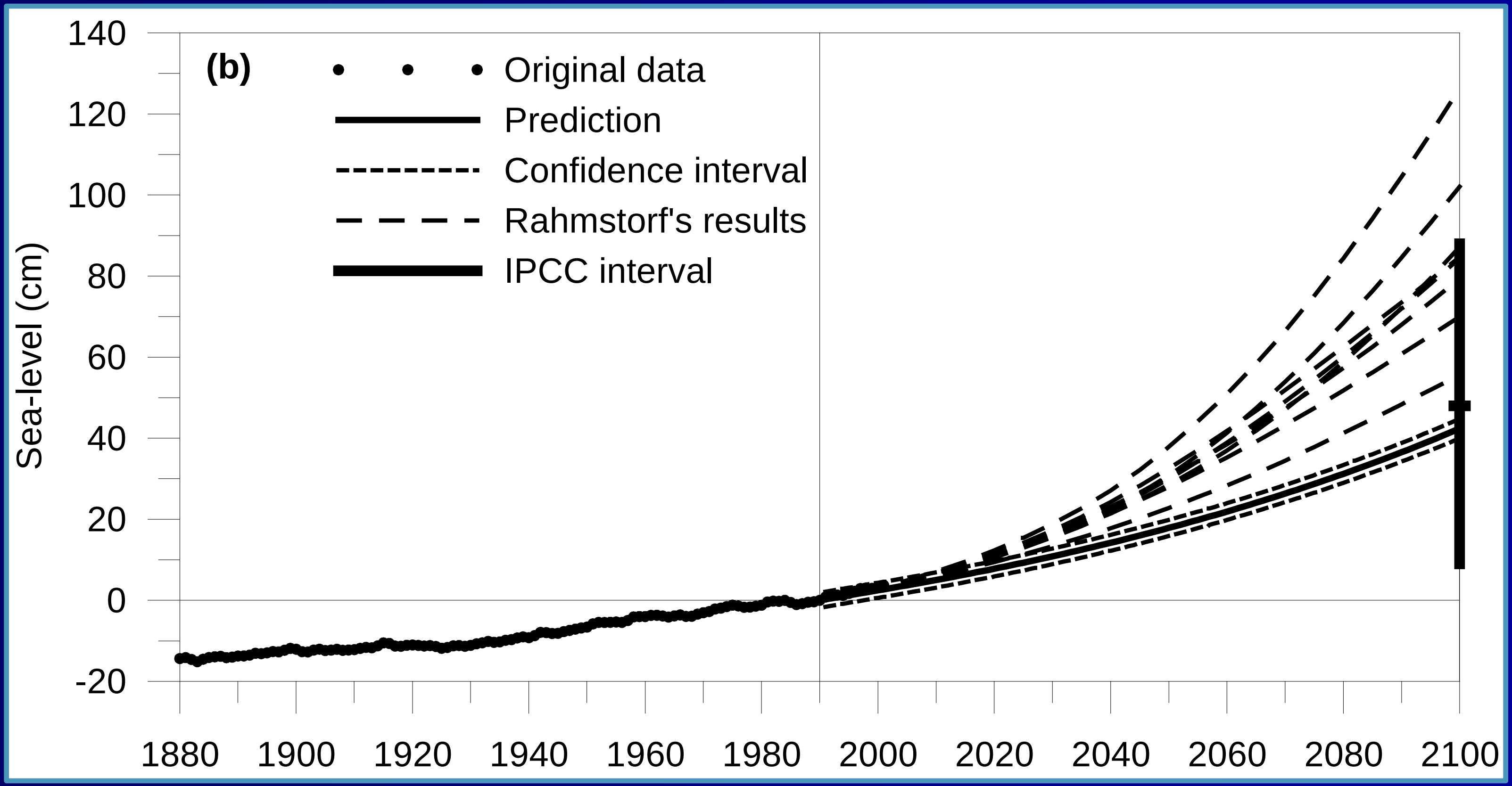
Temperature (1990-2100)

with IPCC scenarios:



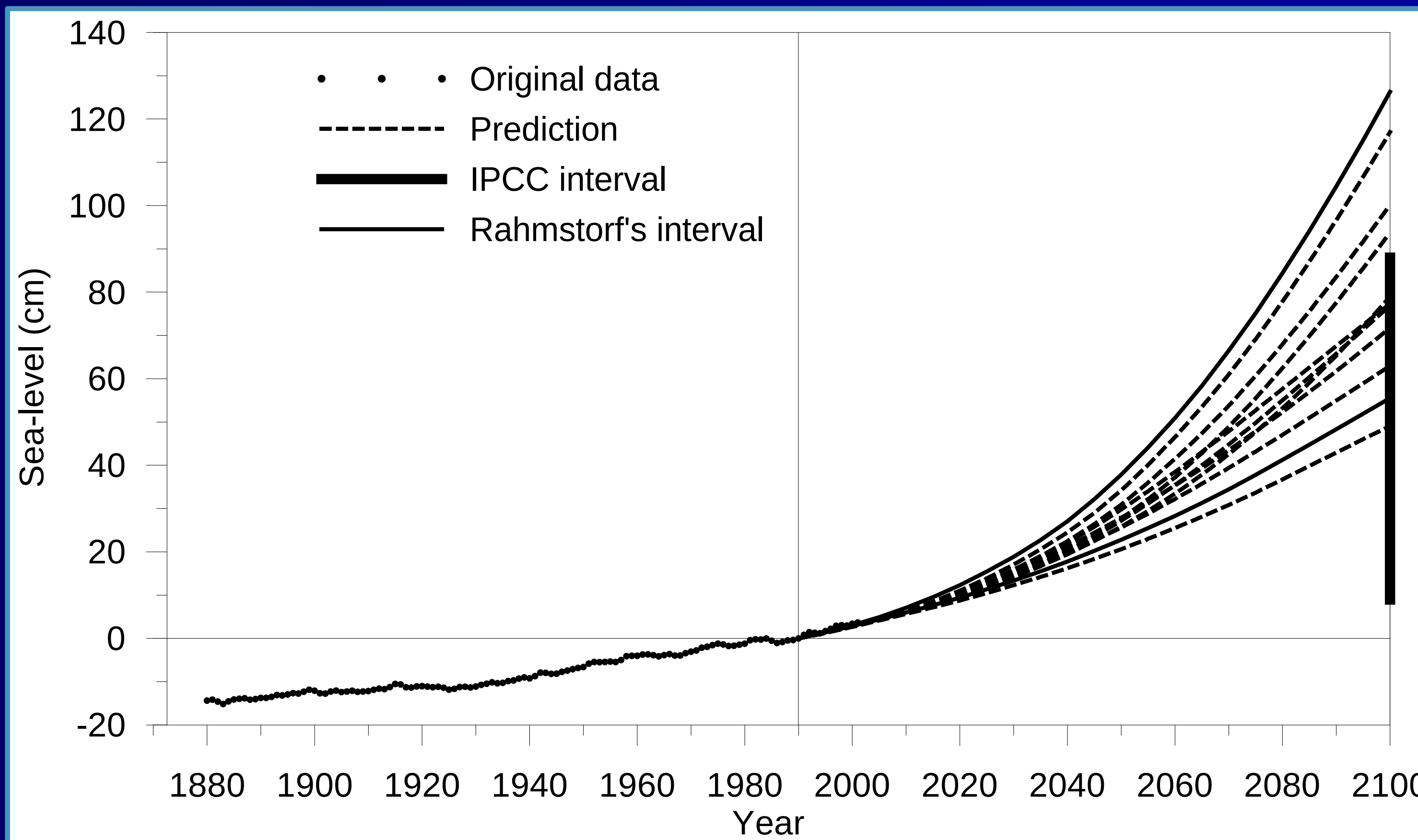
Sea-Level (1990-2100)

with IPCC scenarios:



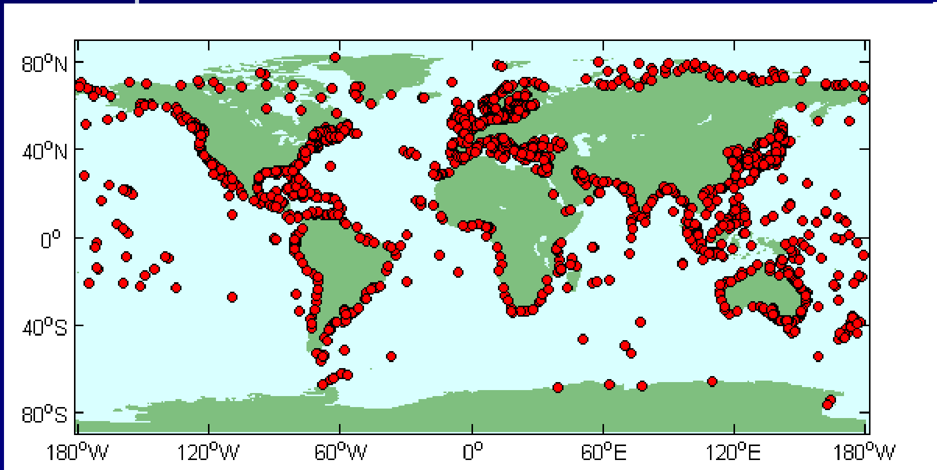
Sea-Level (1990-2100)

with IPCC scenarios:

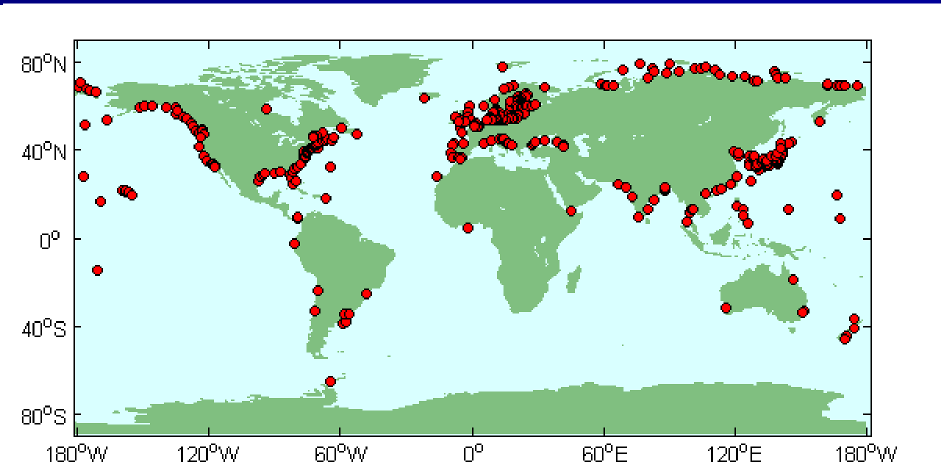


Sea level records—tide gauges

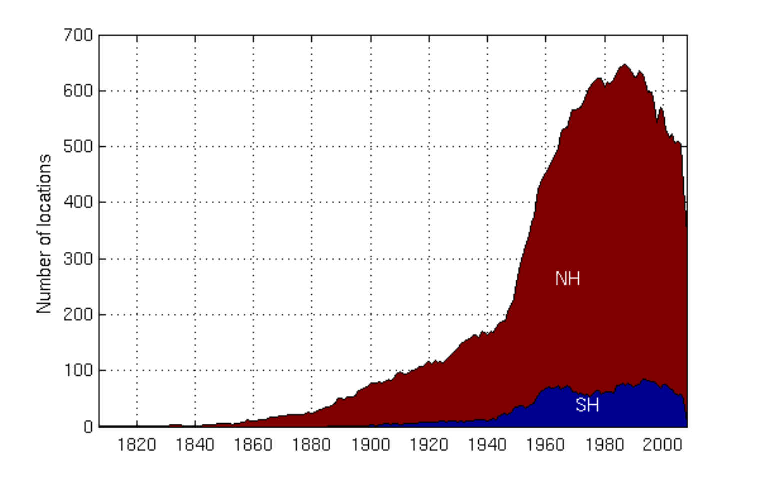
- Permanent Service for Mean Sea Level (PSMSL)



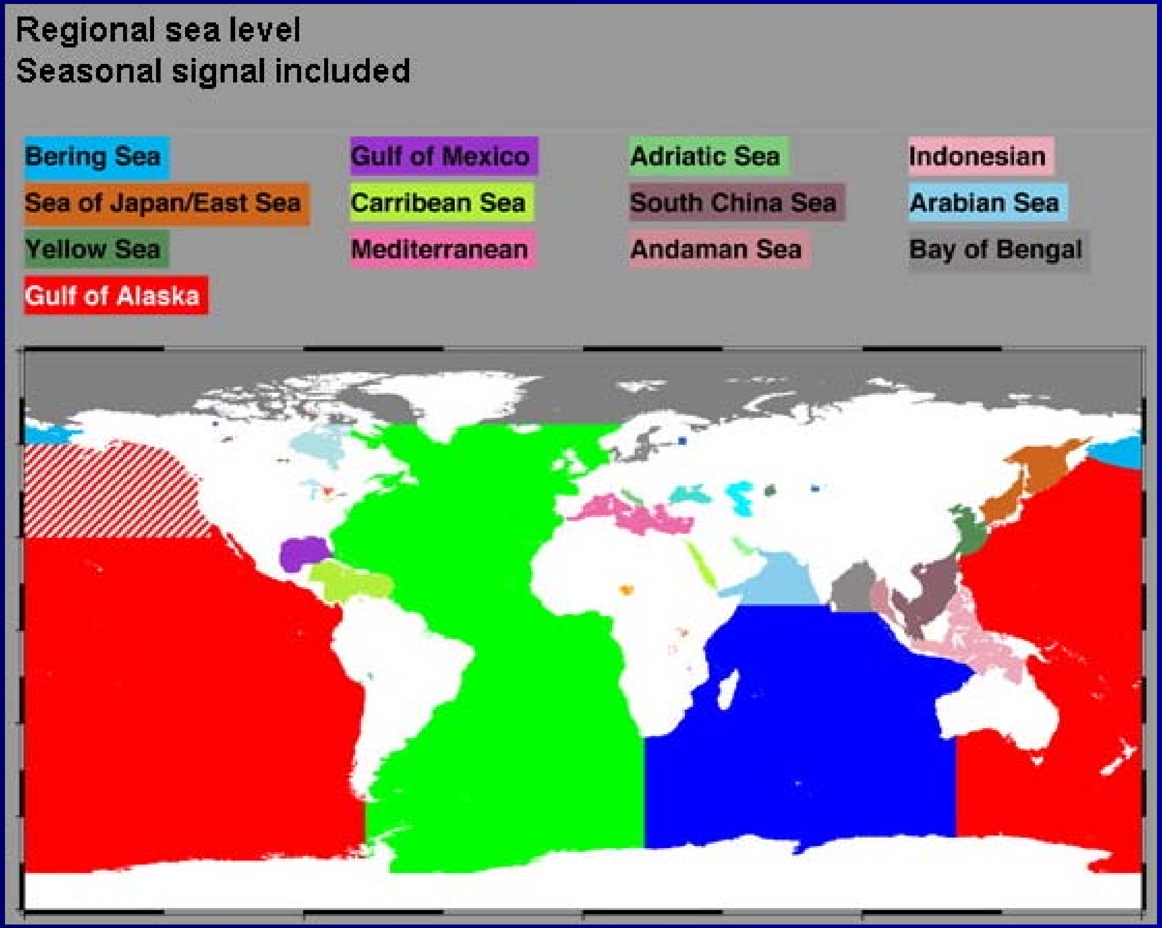
Distribution of PSMSL stations



PSMSL stations with at least 40 years of RLR data



Advantages: Spatial Analysis



Advantages: Spatial Analysis

$$\left\{\begin{matrix} \dfrac{dT_1(t)}{dt} \\ \dfrac{dT_2(t)}{dt} \\ . \\ \dfrac{dT_n(t)}{dt} \end{matrix}\right\} = [A_{11}] \left\{\begin{matrix} T_1(t) \\ T_2(t) \\ . \\ T_n(t) \end{matrix}\right\} + [A_{12}] \left\{\begin{matrix} H_1(t) \\ H_2(t) \\ . \\ H_n(t) \end{matrix}\right\} + \left\{\begin{matrix} C_{11} \\ C_{12} \\ . \\ C_{1n} \end{matrix}\right\}$$

$$\left\{\begin{matrix} \dfrac{dH_1(t)}{dt} \\ \dfrac{dH_2(t)}{dt} \\ . \\ \dfrac{dH_n(t)}{dt} \end{matrix}\right\} = [A_{21}] \left\{\begin{matrix} T_1(t) \\ T_2(t) \\ . \\ T_n(t) \end{matrix}\right\} + [A_{22}] \left\{\begin{matrix} H_1(t) \\ H_2(t) \\ . \\ H_n(t) \end{matrix}\right\} + \left\{\begin{matrix} C_{21} \\ C_{22} \\ . \\ C_{2n} \end{matrix}\right\}$$

Advantages: Forcing Function(s)

$$\frac{dT(t)}{dt} = a_{11}T(t) + a_{12}H(t) + a_{13}U_i\left(t\right) + c_1$$

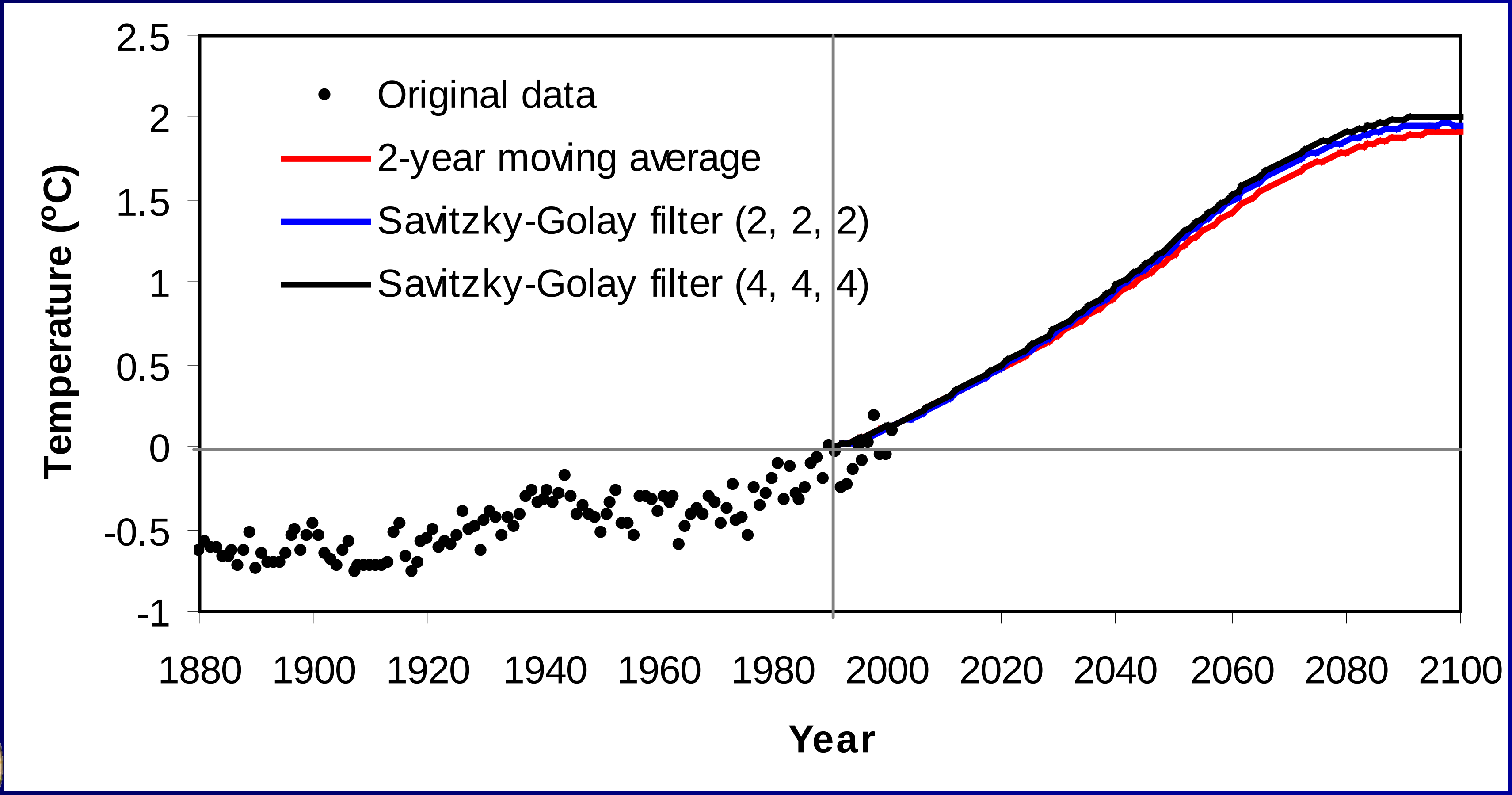
$$\frac{dH(t)}{dt} = a_{21}T(t) + a_{22}H(t) + a_{23}U_i\left(t\right) + c_2$$

Spatial Analysis & Forcing Function(s):

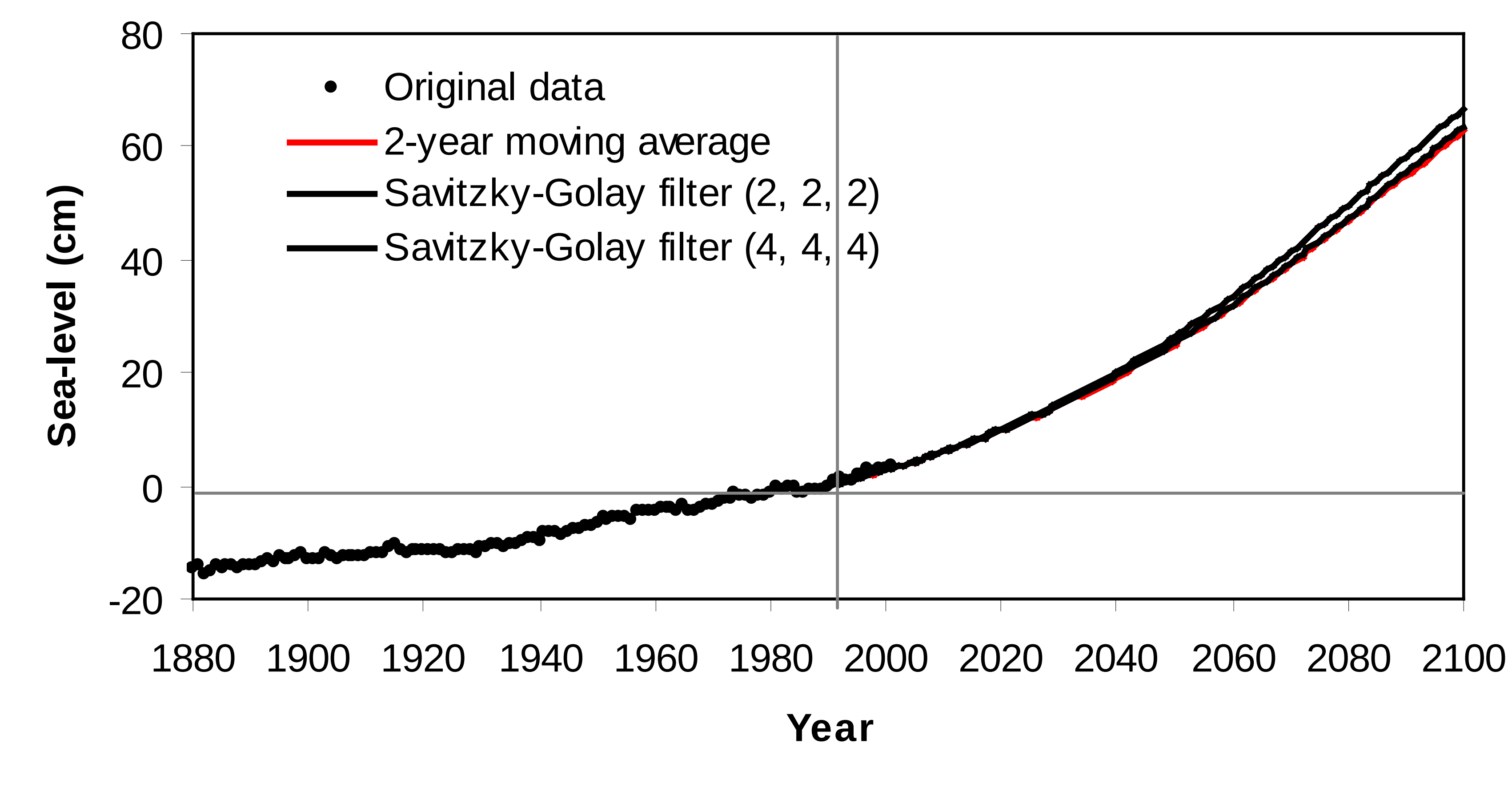
$$\begin{Bmatrix} \frac{dT_1(t)}{dt} \\ \frac{dT_2(t)}{dt} \\ . \\ \frac{dT_n(t)}{dt} \end{Bmatrix} = [A_{11}] \begin{Bmatrix} T_1(t) \\ T_2(t) \\ . \\ T_n(t) \end{Bmatrix} + [A_{12}] \begin{Bmatrix} H_1(t) \\ H_2(t) \\ . \\ H_n(t) \end{Bmatrix} + [A_{13}] \begin{Bmatrix} U_1(t) \\ U_2(t) \\ . \\ U_n(t) \end{Bmatrix} + \begin{Bmatrix} C_{11} \\ C_{12} \\ . \\ C_{1n} \end{Bmatrix}$$

$$\begin{Bmatrix} \frac{dH_1(t)}{dt} \\ \frac{dH_2(t)}{dt} \\ . \\ \frac{dH_n(t)}{dt} \end{Bmatrix} = [A_{21}] \begin{Bmatrix} T_1(t) \\ T_2(t) \\ . \\ T_n(t) \end{Bmatrix} + [A_{22}] \begin{Bmatrix} H_1(t) \\ H_2(t) \\ . \\ H_n(t) \end{Bmatrix} + [A_{23}] \begin{Bmatrix} U_1(t) \\ U_2(t) \\ . \\ U_n(t) \end{Bmatrix} + \begin{Bmatrix} C_{21} \\ C_{22} \\ . \\ C_{2n} \end{Bmatrix}$$

Spatial Analysis & Forcing Function(s) Global CO_2 Impact:



Spatial Analysis & Forcing Function(s) Global CO_2 Impact:



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