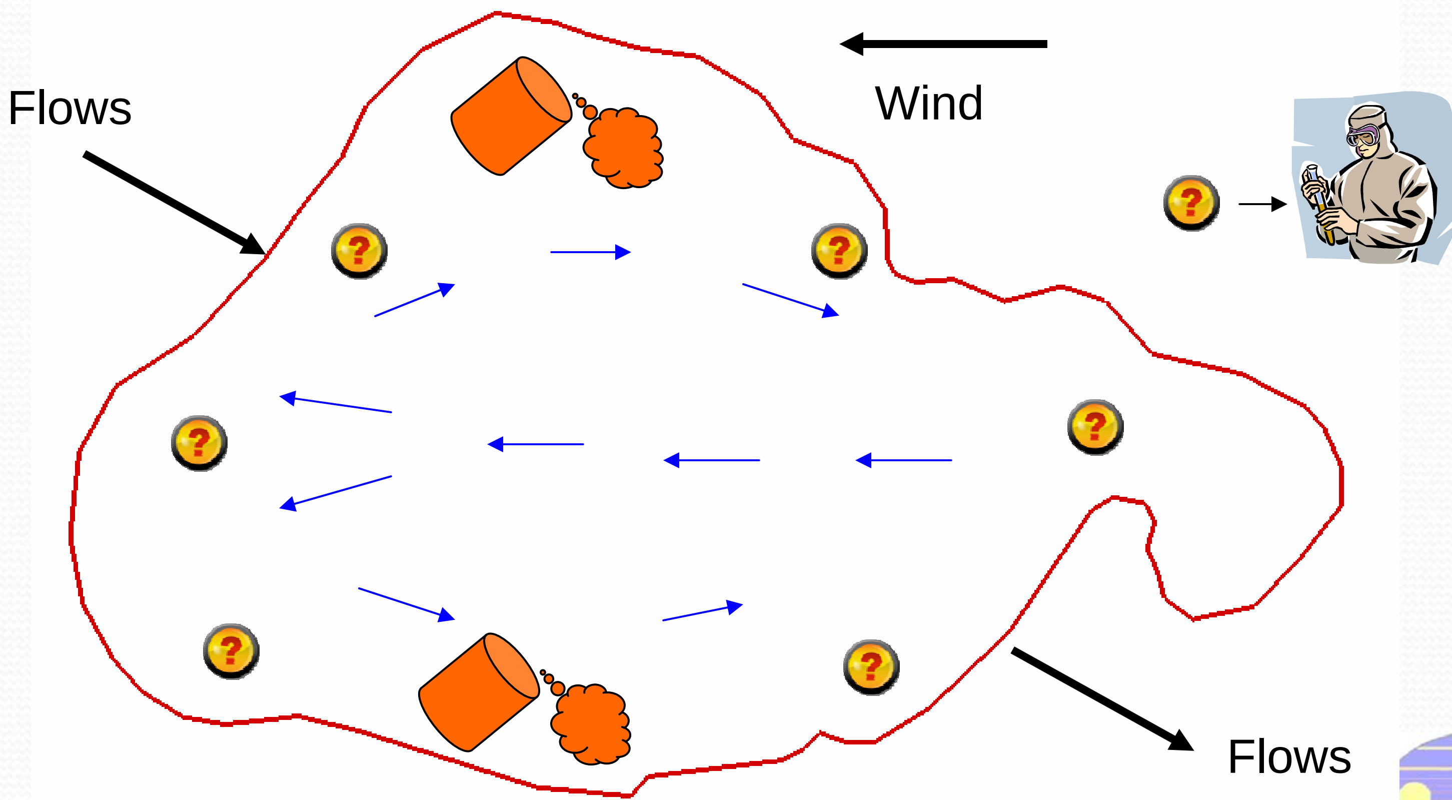
Optimal Sensor Placement for Wind-Driven Lakes

Kijin Nam and Mustafa M. Aral
Multimedia Environmental Simulation Lab (MESL)
(http://mesl.ce.gatech.edu/)
School of Civil and Environmental Engineering
Georgia Institute of Technology

Needs to design a water quality monitoring system

- Requirement of water quality monitoring increases due to increasing health risk from degrading water quality.
  - Setting up and running a monitoring network is very costly and the performance depends on its design.
    - Monitoring locations are one of the most important factors.
  - However, a clear strategy to design a water quality monitoring network does not exist.

Where are the best monitoring locations? And How many?



Optimization of Sensor Locations
- Objective

- Minimize detection time
  - Early detection allows more time to respond and helps to maximize usability of water body.
- Decision variables
  - Sensor locations
- Objective function

$$\min f(X) = \frac{1}{S_n} \sum_{i=1}^{S_n} t_d^{s_i}(X, t_r^{s_i}, \mathbf{x}_r^{s_i})$$

$t_d^{s_i}(X, t_r^{s_i}, \mathbf{x}_r^{s_i})$: Detection time of scenario s_i

$t_r^{s_i}$: Time of contaminant release of scenario s_i

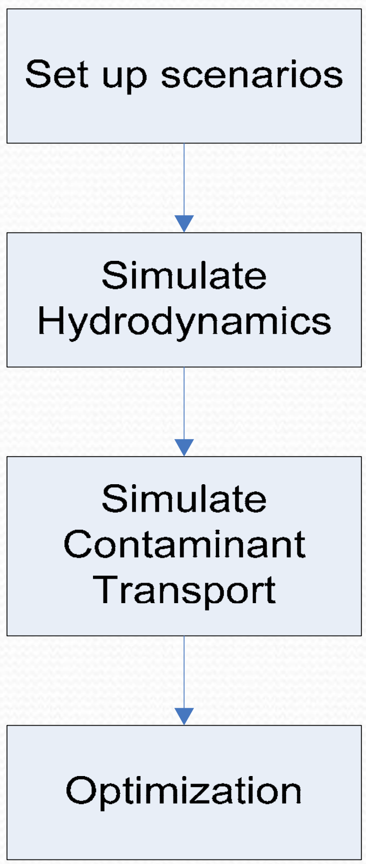
$\mathbf{x}_r^{s_i}$: Location of contaminant release of scenario s_i

$\mathbf{x}_i$: Location of a sensor

$X = \{\mathbf{x}_1, \ldots, \mathbf{x}_i, \ldots, \mathbf{x}_n\}$: Sensor distribution

S_n : Number of all scenarios

Procedure



- What hydrodynamics and contaminant source conditions we have
    - Forcing for hydrodynamics
    - Source locations, release times
- Simulate every hydrodynamics scenario using the finite element method
- Simulate every contaminant transport scenario using the finite element method
- Calculate detection time at possible sensor locations
- Optimize sensor locations to minimize detection time using the genetic algorithm

Simulation of Physics

- Shallow water simulation

$$\frac{\partial h}{\partial t}+\frac{\partial (hU)}{\partial x}+\frac{\partial (hV)}{\partial y}=0$$

$$\frac{\partial (hU)}{\partial t}+\frac{\partial (hUU)}{\partial x}+\frac{\partial (hVU)}{\partial y}-E_H\left(\frac{\partial^2 (hU)}{\partial x^2}+\frac{\partial^2 (hU)}{\partial y^2}\right)+gh\frac{\partial \eta}{\partial x}-\tau_{sx}+\tau_{bx}=0$$

$$\frac{\partial (hV)}{\partial t}+\frac{\partial (hUV)}{\partial x}+\frac{\partial (hVV)}{\partial y}-E_H\left(\frac{\partial^2 (hV)}{\partial x^2}+\frac{\partial^2 (hV)}{\partial y^2}\right)+gh\frac{\partial \eta}{\partial y}-\tau_{sy}+\tau_{by}=0$$

- Contaminant transport simulation

$$\frac{\partial (hC)}{\partial t}+\frac{\partial (UhC)}{\partial x}+\frac{\partial (VhC)}{\partial y}$$
$$=\frac{\partial}{\partial x}\left(D_{xx}\frac{\partial (hC)}{\partial x}\right)+\frac{\partial}{\partial x}\left(D_{xy}\frac{\partial (hC)}{\partial y}\right)+\frac{\partial}{\partial y}\left(D_{yx}\frac{\partial (hC)}{\partial x}\right)+\frac{\partial}{\partial y}\left(D_{yy}\frac{\partial (hC)}{\partial y}\right)$$

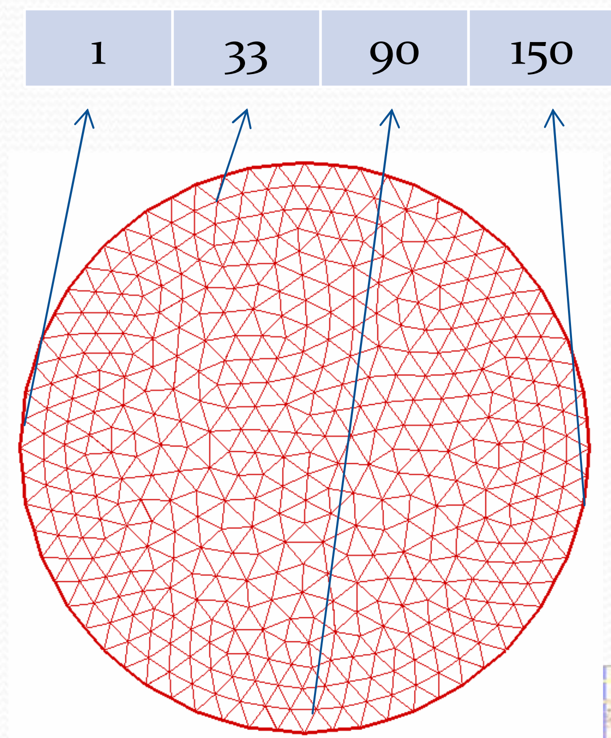
- Finite Element Method on a parallel computing environment

Optimization – Genetic Algorithm

- Genetic algorithm
  - Suitable for complex objective functions
    - Does not require any gradient calculation
    - Does not fall into local optima easily
- Genetic algorithm can be parallelized
  - Island model
    - Helps to keep diversity
- The optimization routine searches the best nodes instead of *x* and *y*-coordinate.

Chromosome and Evaluation

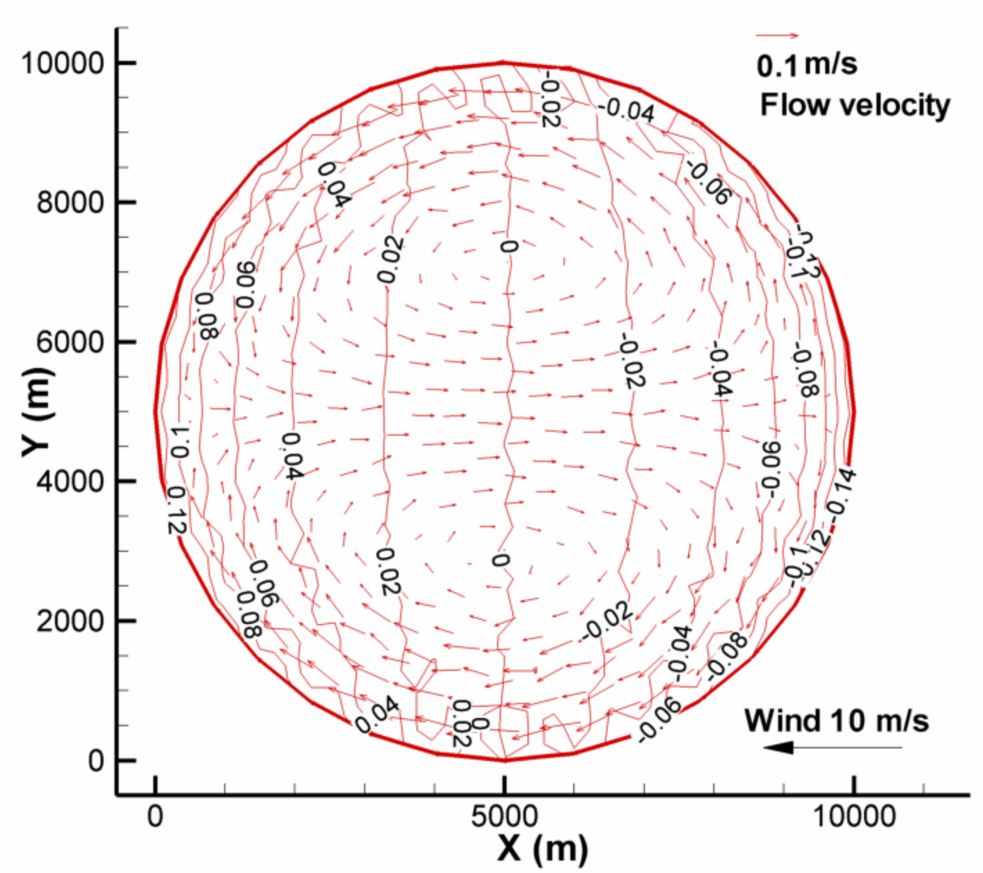
- Each gene represents a nodal index in a mesh, where a sensor is placed.
  - Simpler and more straight-forward than a bit-string
    - The problem becomes an integer programming.
- Evaluation of fitness: Calculate how long it takes for a contaminant to reach to one of sensors.



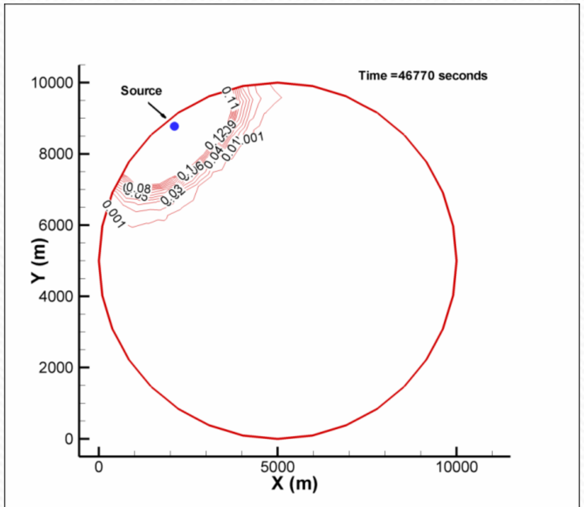
Test Problem - Circular Lake

- To test the ability of the proposed approach, a circular lake with a wind-driven flow is set up and tested.
  - Deep at the center, shallow along the shoreline
  - Wind from the east
  - Flow reaches to a steady state.
  - Elements on the shoreline are regarded as possible source locations.

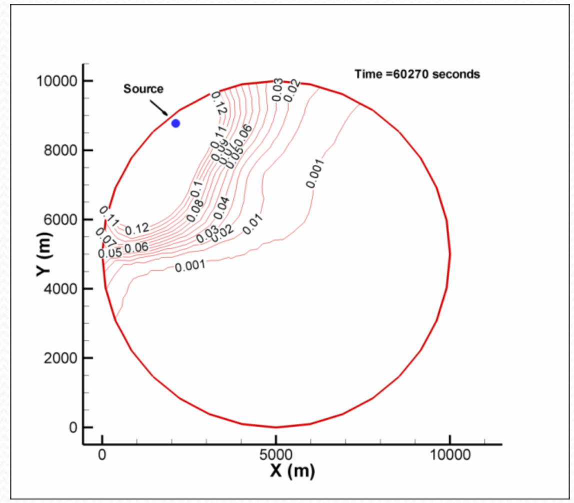
Circular Lake – Flow Pattern



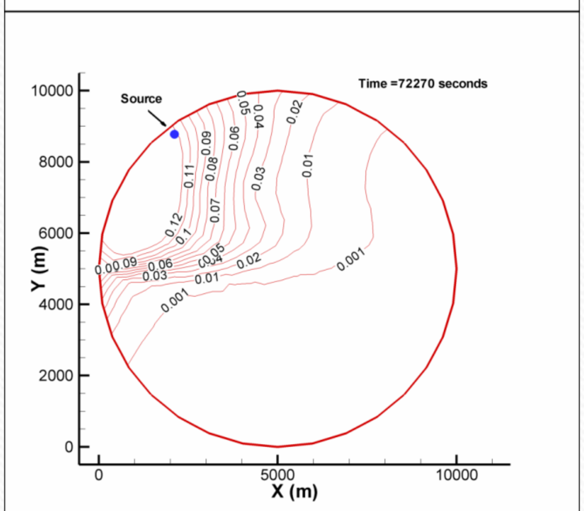
Circular Lake – Contaminant Transport



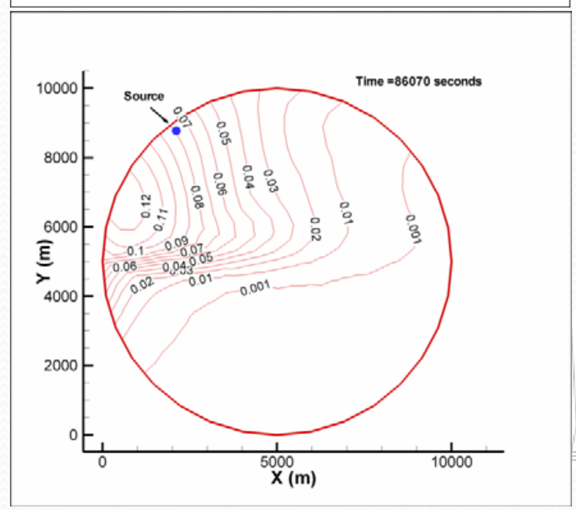
1 hr



4 hrs 40 min

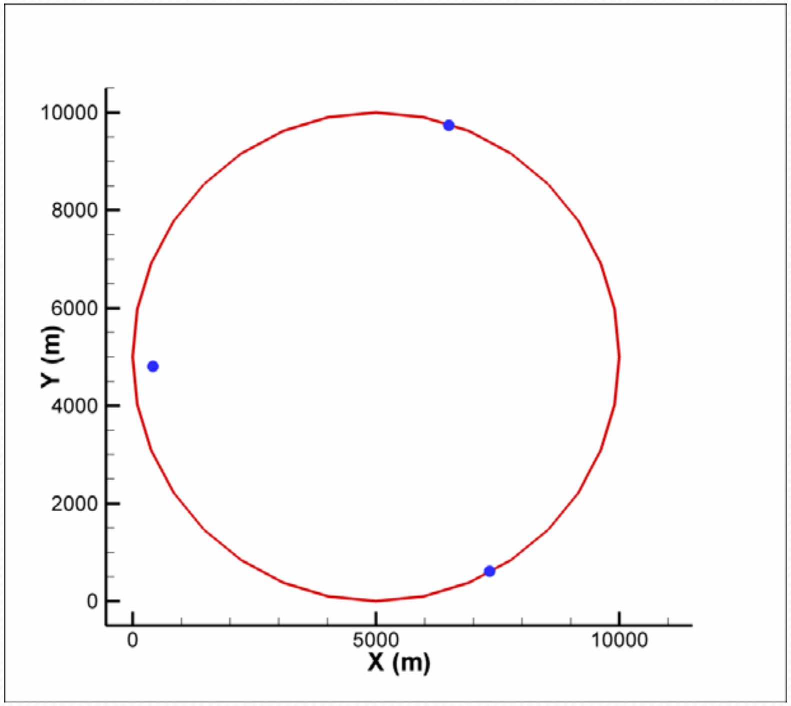


8 hrs

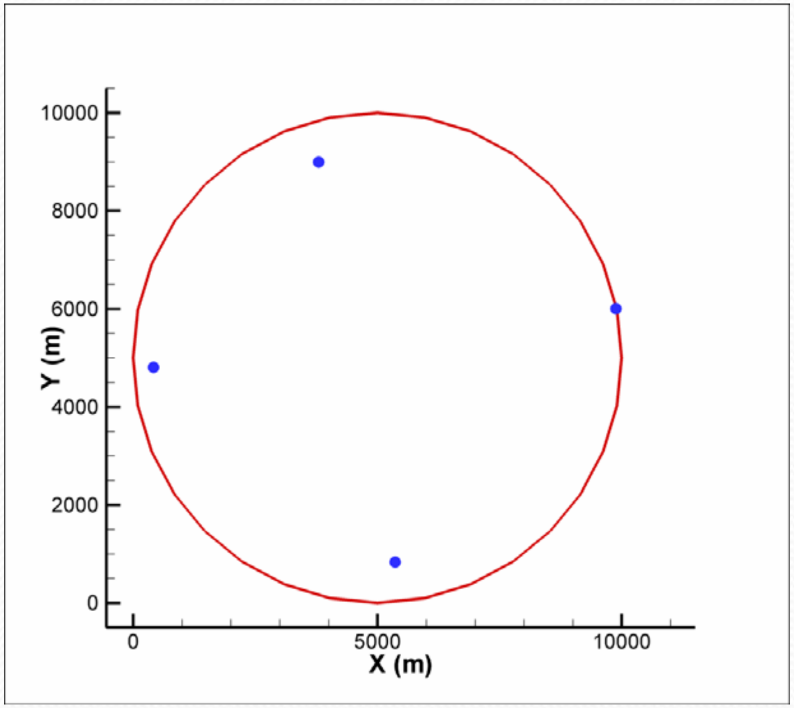


12 hrs

Circular Lake – Optimized Locations



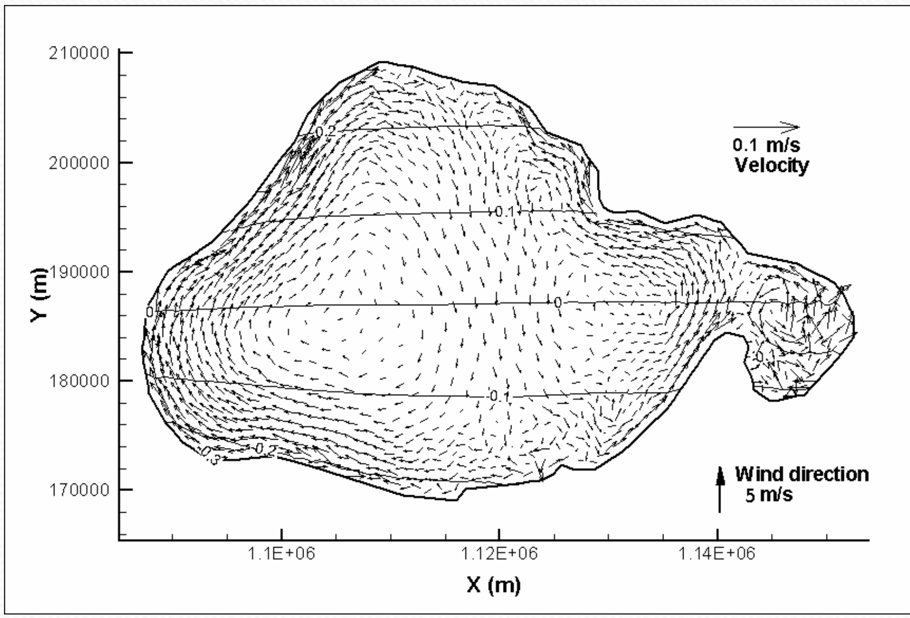
3 sensors

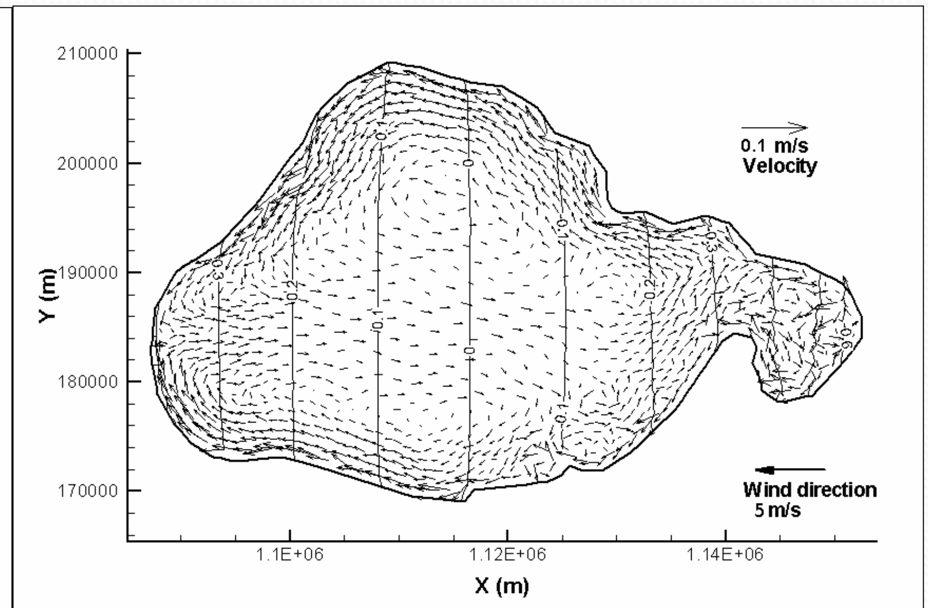


4 sensors

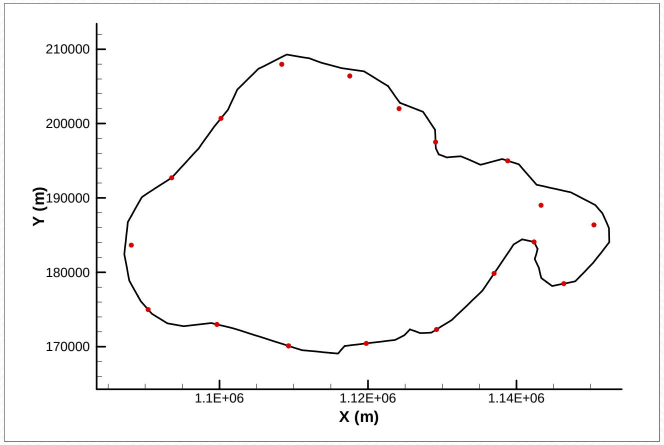
Lake Pontchartrain – Flow Pattern

- Similar setting as the circular lake case

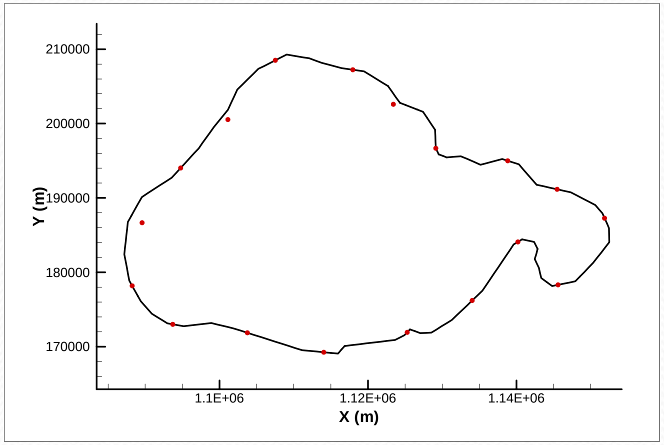




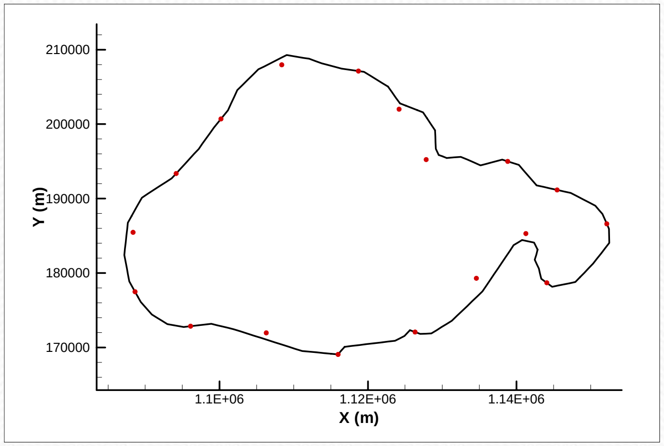
Lake Pontchartrain – Optimized Locations



Wind from the south



Wind from the east



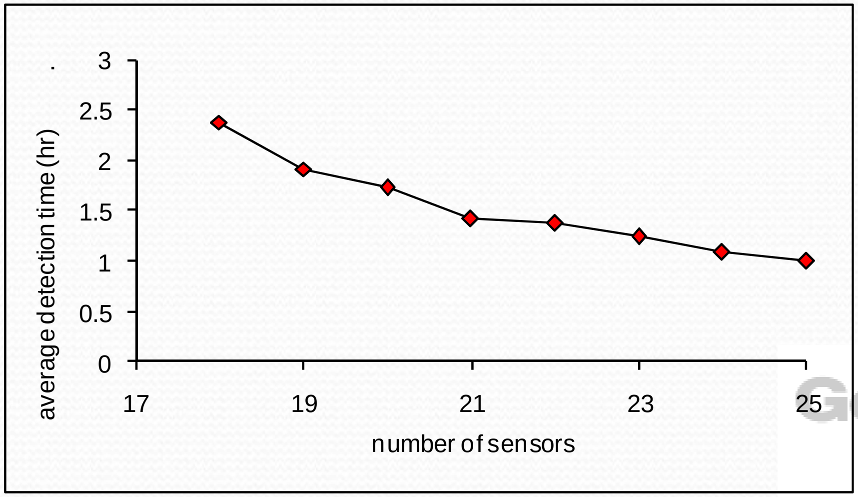
Wind from the south and east

- Evenly distributed along the shoreline regardless of wind direction

Lake Pontchartrain

– Average detection time vs. # of sensors

- As the number of sensor increases, the average detection time decreases.
  - Provide trade-off between detection time and the number of sensors (= cost)



Conclusions

- Locations of monitoring sensors are critical factors of the performance of monitoring sensors.
- The Combination of numerical simulation and genetic algorithm shows promising results on a realistic way.
  - This approach may provide valuable information for decision makers to set up a monitoring system.
- In wind-driven lakes with a steady-state, best sensor locations will be places along a shoreline.

Current Work

- Finding a best path to measure, instead of several fixed sensor locations
  - Measurement using a remote-controlled boat may be an easier and dynamic solution.
- Optimization of Multiple objectives
  - # of sensors
  - What part of a domain should be protected more
- Unsteady state problems are being solved.