Numerical simulation of two-phase Darcy-Forchheimer flow during CO_2 injection into deep saline aquifers

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Darcy flow VS non-Darcy flow

- Darcy flow

A linear relationship between volumetric flow rate (Darcy velocity) and pressure (or potential) gradient

Dominant at low flow rates

$$-\nabla\Phi = \frac{\mu v}{k}$$

Φ is the flow potential;

μ is the viscosity;

v is the Darcy velocity;

k is the intrinsic permeability

- Non-Darcy flow

Any deviations from the linear relation may be defined as non-Darcy flow

Interested in the nonlinear relationship that accounts for the extra friction or inertial effects at high pressure gradients/ high velocity

Non-Darcy flow equations

Forchheimer equation

$$-\nabla\Phi = \frac{\mu}{k}\,\mathrm{v} + \beta\rho\mathrm{v}|\mathrm{v}|$$

β is the non-Darcy flow coefficient, or Forchheimer coefficient (Forchheimer, 1901)

Baree and Conway equation

$$-\nabla\Phi = \frac{\mu\,\mathrm{v}}{k_d\left(k_{mr} + \dfrac{(1-k_{mr})\mu\,\tau}{\mu\tau + \rho|\mathrm{v}|}\right)}$$

(Baree and Conway, 2004 and 2007)

k_d is absolute Darcy permeability;

k_{mr} is the minimum permeability ratio at high flow rate;

τ is the the characteristic length

Darcy-Forchheimer flow

- Darcy-Forchheimer flow is defined as the flow incorporating the transition between Darcy and Forchheimer flows

Transition criteria

- The Reynolds number (Type-I)

applied mainly in the cases where the representative particle diameter is available

$$\mathrm{Re} = \frac{\rho d v}{\mu}$$

d is the diameter of particles

- The Forchheimer number (Type-II)

used mainly in numerical models

$$f = \frac{\rho k \beta v}{\mu}$$

consistent definition
physical meaning of the variables

Research objectives

- Develop a generalized Darcy-Forchheimer model
- Propose a method to determine the critical Forchheimer number for single and multiphase flows
- Use the model and method to analyze the Darcy-forchheimer flow in the near well-bore area during CO_2 injection into DSA

Math model for two-phase flow

$$\frac{\partial(\theta\rho_\alpha S_\alpha)}{\partial t} = -\nabla\bullet(\rho_\alpha v_\alpha) + Q_\alpha \qquad ; \ \alpha = \ w, \ n$$

θ : porosity;

S_α : saturation;

Q_α : source and sink.

$$-\frac{dp_\alpha}{dx} = \frac{\mu_\alpha v_\alpha}{k\ k_r^\alpha} + \beta_\alpha\rho_\alpha v_\alpha |v_\alpha| \qquad ; \ \alpha = \ w, \ n$$

$$f_\alpha = \frac{kk_r^\alpha}{\mu_\alpha}\beta_\alpha\rho_\alpha|v_\alpha| \qquad ; \quad \alpha = w, \ n$$

$$v_\alpha = \ -(\frac{k_r^\alpha k}{\mu_\alpha}\frac{1}{1+f_\alpha}(\frac{dp_\alpha}{dx})) \qquad ; \ \alpha = \ w, \ n$$

$$\frac{\partial(\theta\rho_\alpha S_\alpha)}{\partial t} = \nabla\bullet(\rho_\alpha\frac{k_r^\alpha k}{\mu_\alpha}\frac{1}{1+f_\alpha}(\nabla p_\alpha)) + Q_\alpha \quad ; \quad \alpha = \ w, \ n$$

Constitutive equations needed

$$S_w + S_n = 1$$

$$P_c = P_n - P_w$$

P_c : capillary pressure;

P_D : entry pressure;

S_w^r : irreducible saturation for water;

S_w^n: irreducible saturation for non-wetting phase;

λ : pore size distribution index.

Brooks-Corey equations :

$$P_c = P_D S_{eff}^{-(1/\lambda)}$$

$$S_{eff} = \frac{S_w - S_w^r}{1 - S_w^r - S_n^r}$$

$$k_r^w = (S_{eff})^{(2+3\lambda)/\lambda}$$

$$k_r^n = (1 - S_{eff})^2 (1 - (S_{eff})^{(2+3\lambda)/\lambda})$$

The Forchheimer number

$$\beta = \frac{0.005}{(k)^{0.5}(\theta)^{5.5}}$$

$$\beta_n = \frac{0.005}{\left(kk_r^n\right)^{0.5}\left(\theta(1-S_w)\right)^{5.5}} = \frac{0.005}{\left(kk_r^n\right)^{0.5}\left(\theta S_n\right)^{5.5}}$$

(Geertsma, 1974)

$$\beta = \frac{2.923 * 10^{-6}\tau}{(k)(\theta)}$$

τ Is tortuosity

(Liu et al., 1995)

$$\beta_\alpha = \frac{2.923 * 10^{-6}\tau}{\left(kk_r^\alpha\right)(\theta)} \quad ; \ \alpha = w,\ n$$

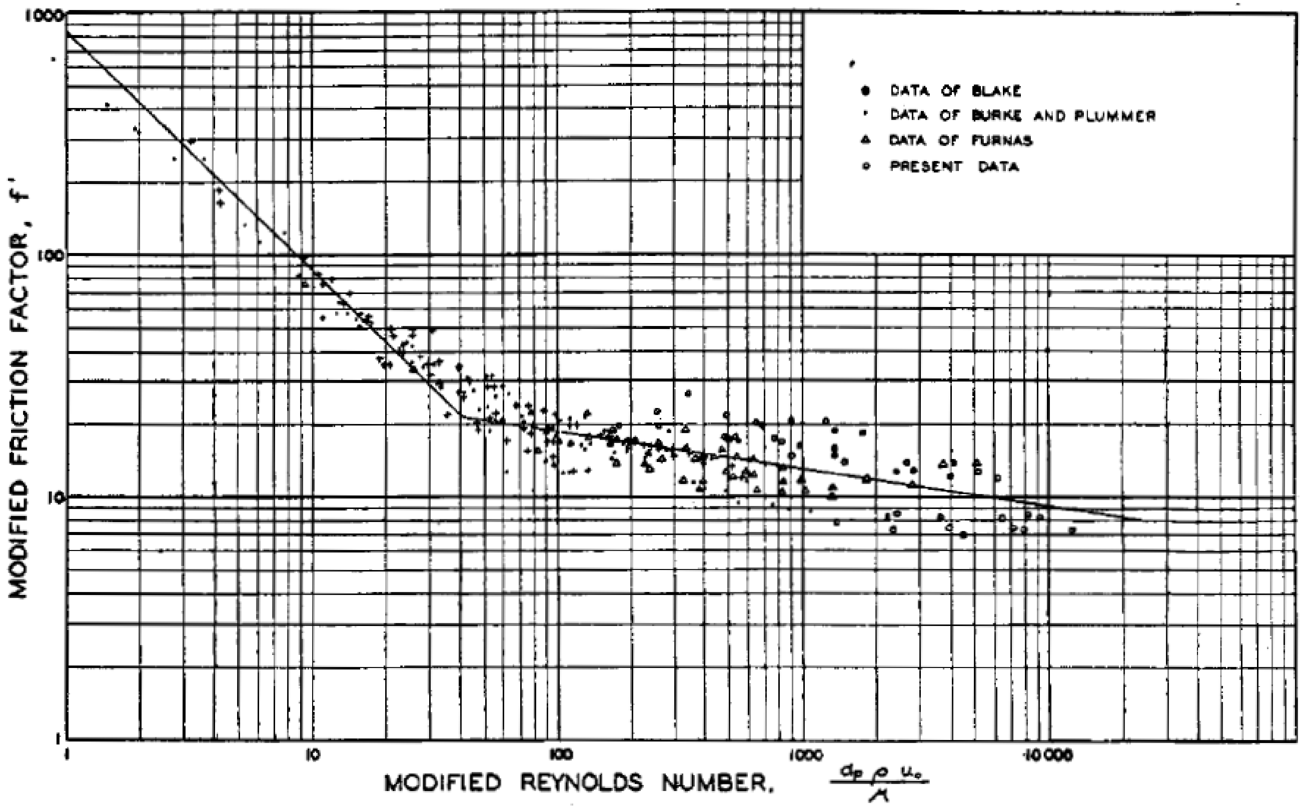
(Ahmadi et al., 2010)

$$\beta_\alpha = \frac{C_\beta \tau}{\left(kk_r^\alpha\right)\left(\theta S_\alpha\right)} \quad ; \ \alpha = w,\ n$$

Evans and Evans (1988) :"a small mobile liquid saturation, such as that occurring in a gas well that also produces water, may increase the non-Darcy flow coefficient by nearly an order of magnitude over that of the dry case."

Determine the critical f_α

Type I: based on Reynolds number for single phase

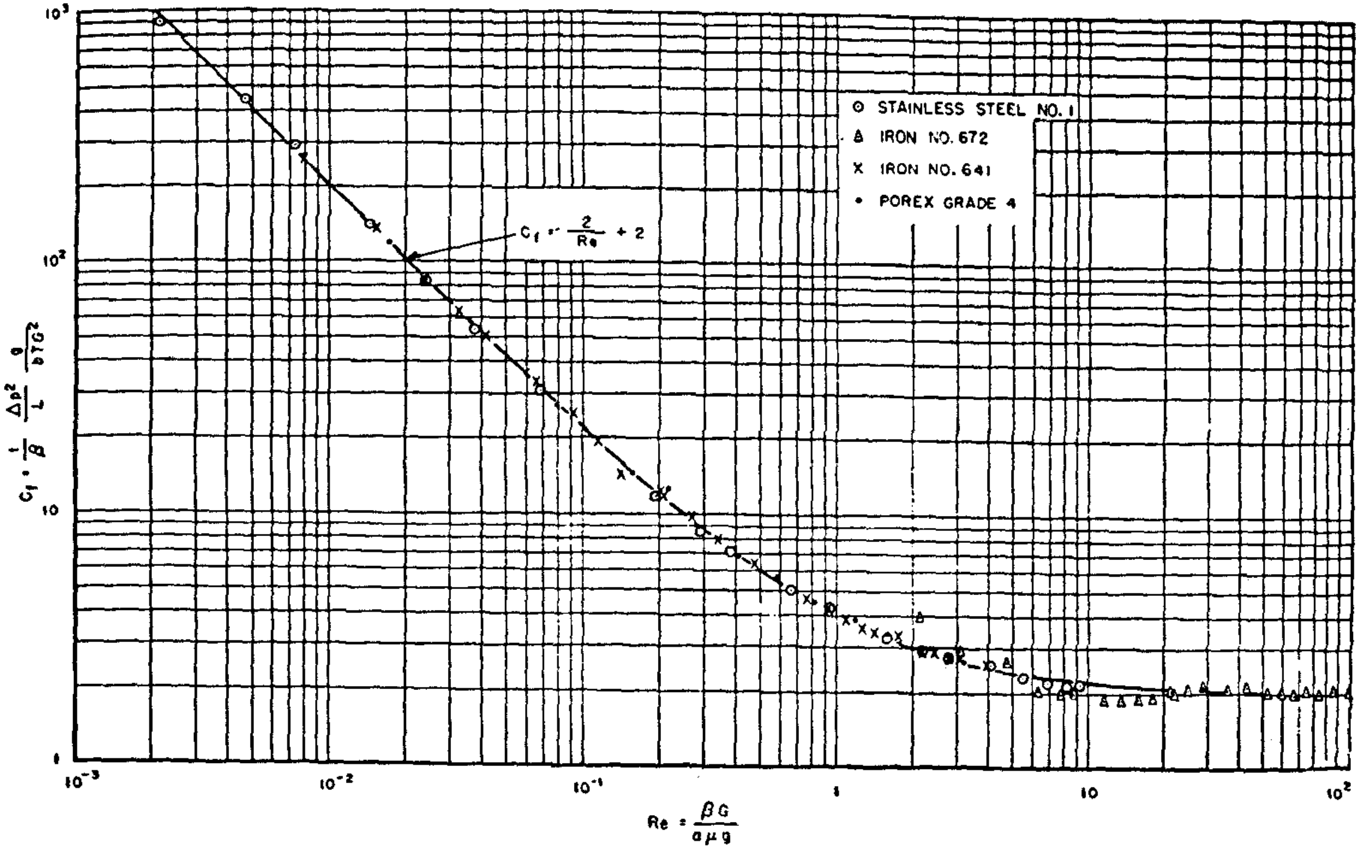


(Chilton et al., 1931)

The point when the linear relationship begins to deviate

Determine the critical f_α

Type II: based on the Forchheimer number for single phase



(Green et al., 1951)

The point when the linear relationship begins to deviate

Intersection of two regression lines

$$Darcy: \quad -\frac{dp}{dx\beta\rho \mathrm{v}^2} = \frac{1}{f}$$

β is not defined in the Darcy formula!!!

$$non-Darcy: \quad -\frac{dp}{dx\beta\rho \mathrm{v}^2} = \frac{1}{f} + 1$$

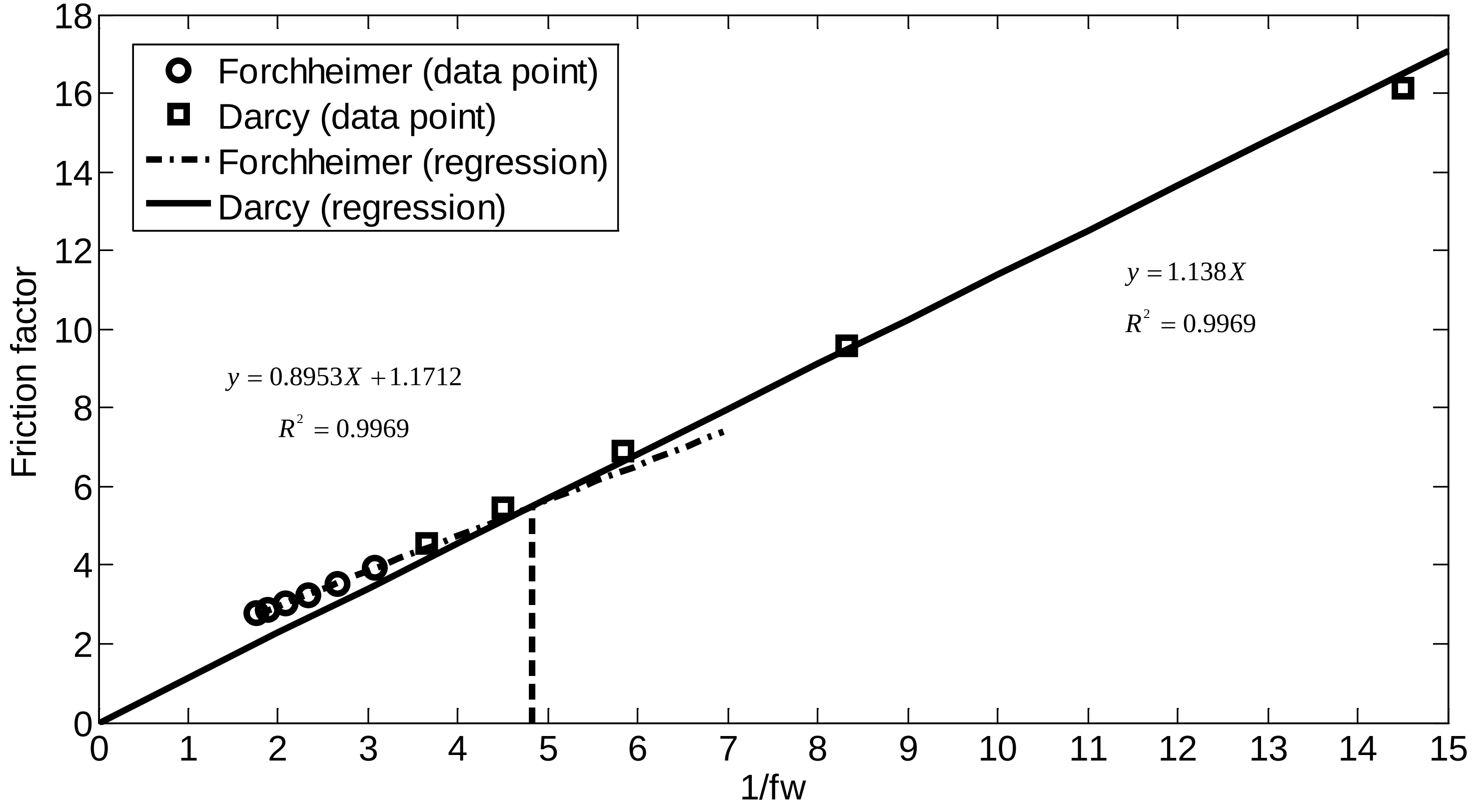
The friction factor is defined as

$$-\frac{dp}{dx\beta\rho v^2}$$

$$Darcy: \quad -\frac{dp_\alpha}{dx\beta_\alpha \rho_\alpha \mathrm{v}_\alpha^2} = \frac{1}{f_\alpha}$$

$$non-Darcy: \quad -\frac{dp_\alpha}{dx\beta_\alpha \rho_\alpha \mathrm{v}_\alpha^2} = \frac{1}{f_\alpha} + 1$$

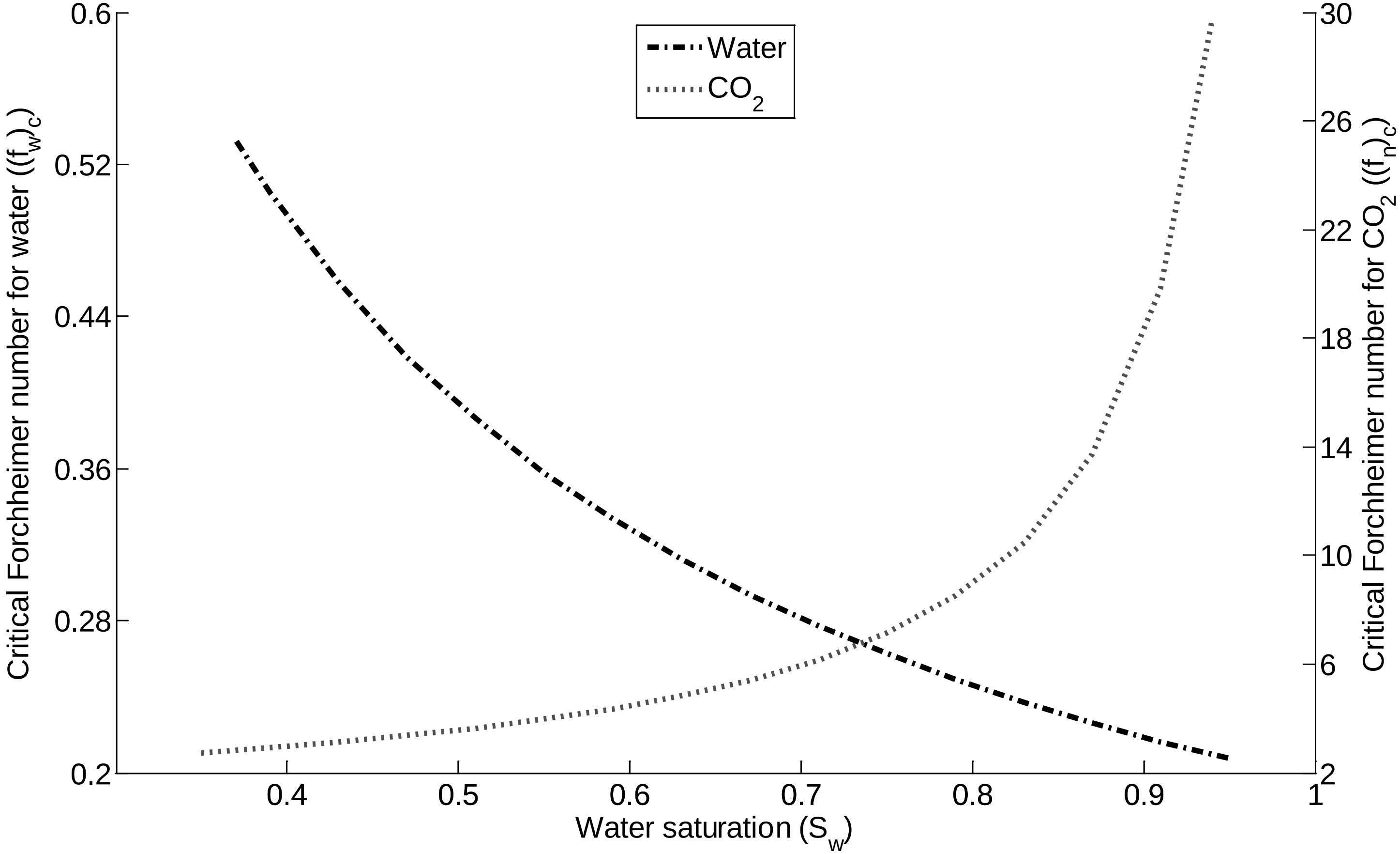
An example plot for 0.95 water saturation



The two lines intercept where 1/fw=4.825, so $\left(f_w\right)_c$ is 1/4.825=0.207

Data from (Sobieski and Trykozko, 2012)

Critical Forchheimer number for H_2O and CO_2 at different saturation values



Numerical model

- Primary variables: P_w and S_n
- Fully-implicit scheme
- Discretization method: CVFD

  Control volume finite difference

Control volume finite difference

For 1D case, the two-phase mass conservation equations can be discretized into

$$\theta S_n^{t+1} + a(p_{wi+1}^{t+1} - p_{wi}^{t+1}) - b(p_{wi}^{t+1} - p_{wi-1}^{t+1}) = \theta S_n^t - \left(\frac{Q_w}{\rho_w}\right)^t \Delta t$$

$$\theta S_n^{t+1} - \left(c(p_{wi+1}^{t+1} - p_{wi}^{t+1}) - d(p_{wi}^{t+1} - p_{wi-1}^{t+1})\right)$$

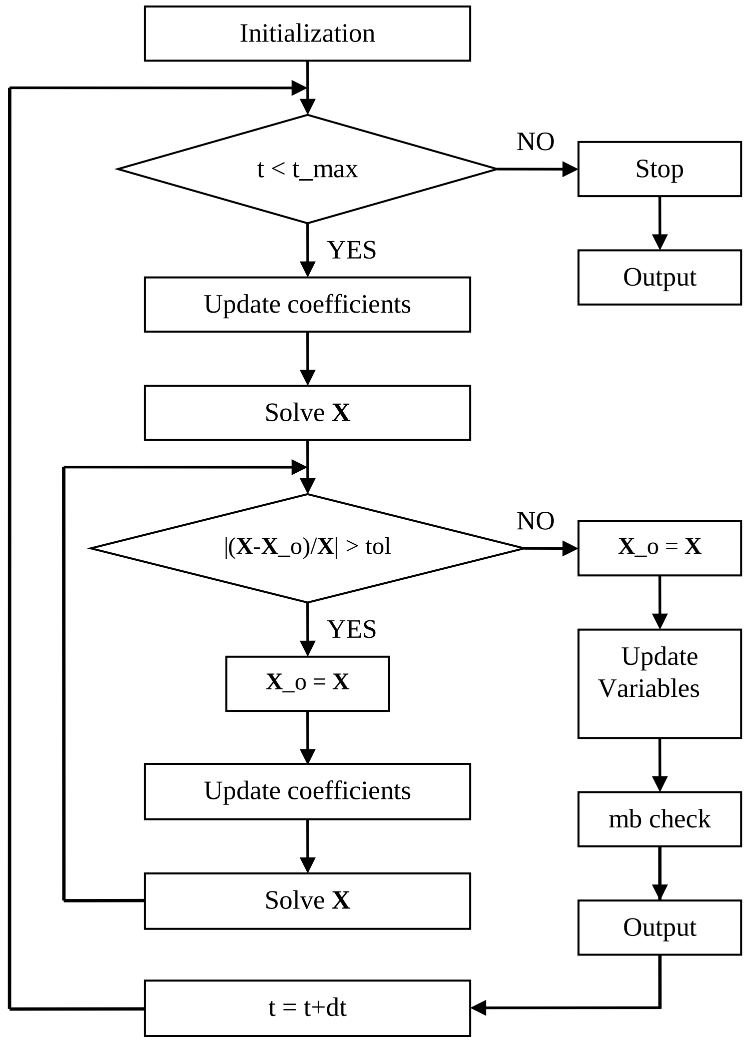
$$-\left(\left(\frac{\partial p_c}{\partial S_n}\right)_{i+\frac{1}{2}} c(S_{ni+1}^{t+1} - S_{ni}^{t+1}) - \left(\frac{\partial p_c}{\partial S_n}\right)_{i-\frac{1}{2}} d(S_{ni}^{t+1} - S_{ni-1}^{t+1})\right) = \theta S_n^t + \left(\frac{Q_n}{\rho_n}\right)^t \Delta t$$

$$a = \left[\frac{\Delta t}{(\Delta x)^2}\frac{kk_r^w}{\mu_w}\frac{1}{1+f_w}\right]_{i+\frac{1}{2}} \qquad b = \left[\frac{\Delta t}{(\Delta x)^2}\frac{kk_r^w}{\mu_w}\frac{1}{1+f_w}\right]_{i-\frac{1}{2}}$$

$$c = \left[\frac{\Delta t}{(\Delta x)^2}\frac{kk_r^n}{\mu_n}\frac{1}{1+f_n}\right]_{i+\frac{1}{2}} \qquad d = \left[\frac{\Delta t}{(\Delta x)^2}\frac{kk_r^n}{\mu_n}\frac{1}{1+f_n}\right]_{i-\frac{1}{2}}$$

$$\mathrm{x} = \left[S_{n1}^{t+1}, \cdots S_{ni}^{t+1}, \cdots S_{nm}^{t+1}, p_{w1}^{t+1} \cdots p_{wi}^{t+1}, \cdots p_{wm}^{t+1}\right]^T \qquad \mathbf{AX} = B$$

Numerical algorithm



For each iteration of each time step, **X** and other related variables in the last time step are used to update all the coefficients including *a*, *b*, *c*, *d*, *dPc/dSn*, *fw*, *fn* and all the elements in the right hand side **B;**

The right hand side term **B** is based on the variables in the last time step and don't need to be updated except for the first iterative step;

Mass balance check

$$I_\alpha = \frac{\sum_{i=1}^{m} V_i \theta [S_{\alpha i}^{\,t} - S_{\alpha i}^{\,t-1}]}{\sum_{i=1}^{m} \Delta t_t (Q_{\alpha i}^{\,t} + \sum_{l \in \Gamma} q_{\alpha l,i}^{\,t})} \quad ; \alpha = w, \; n$$

$$C_\alpha = \frac{\sum_{i=1}^{m} V_i \theta [S_{\alpha i}^{\,t} - S_{\alpha i}^{\,0}]}{\sum_{j=1}^{t} \Delta t_j \sum_{i=1}^{m} (Q_{\alpha i}^{\,j} + \sum_{l \in \Gamma} q_{\alpha l,i}^{\,j})} \quad ; \alpha = w, \; n$$

Γ is the boundaries of the domain
m is the number of the nodes;
t is the number of time steps;
Q is discharge for pumping or injecting wells;
q is the flow rate through the boundaries;

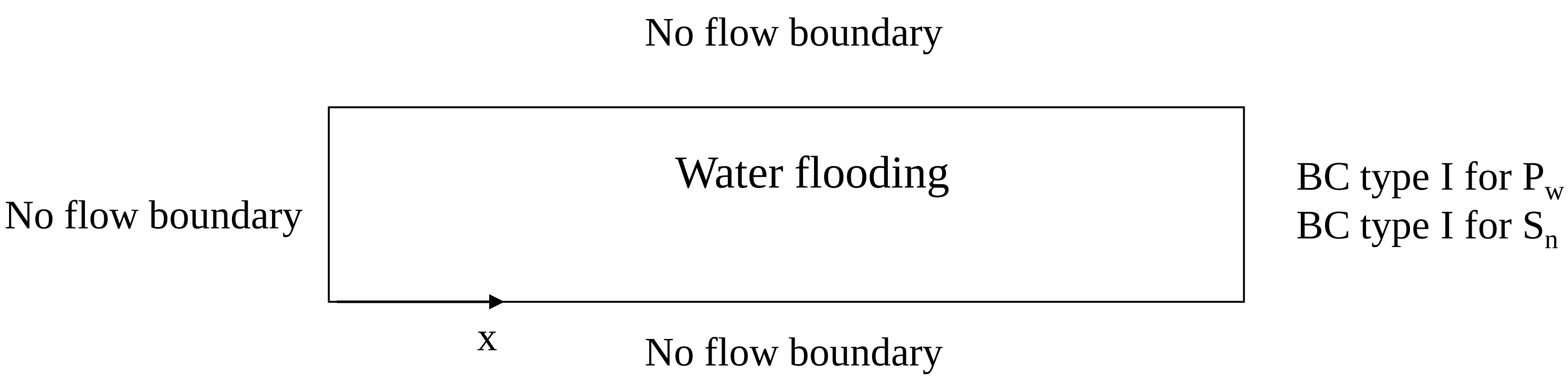
For Q and q, they are set to be positive if entering the domain while negative if leaving the domain.

Darcy-Forchheimer flow

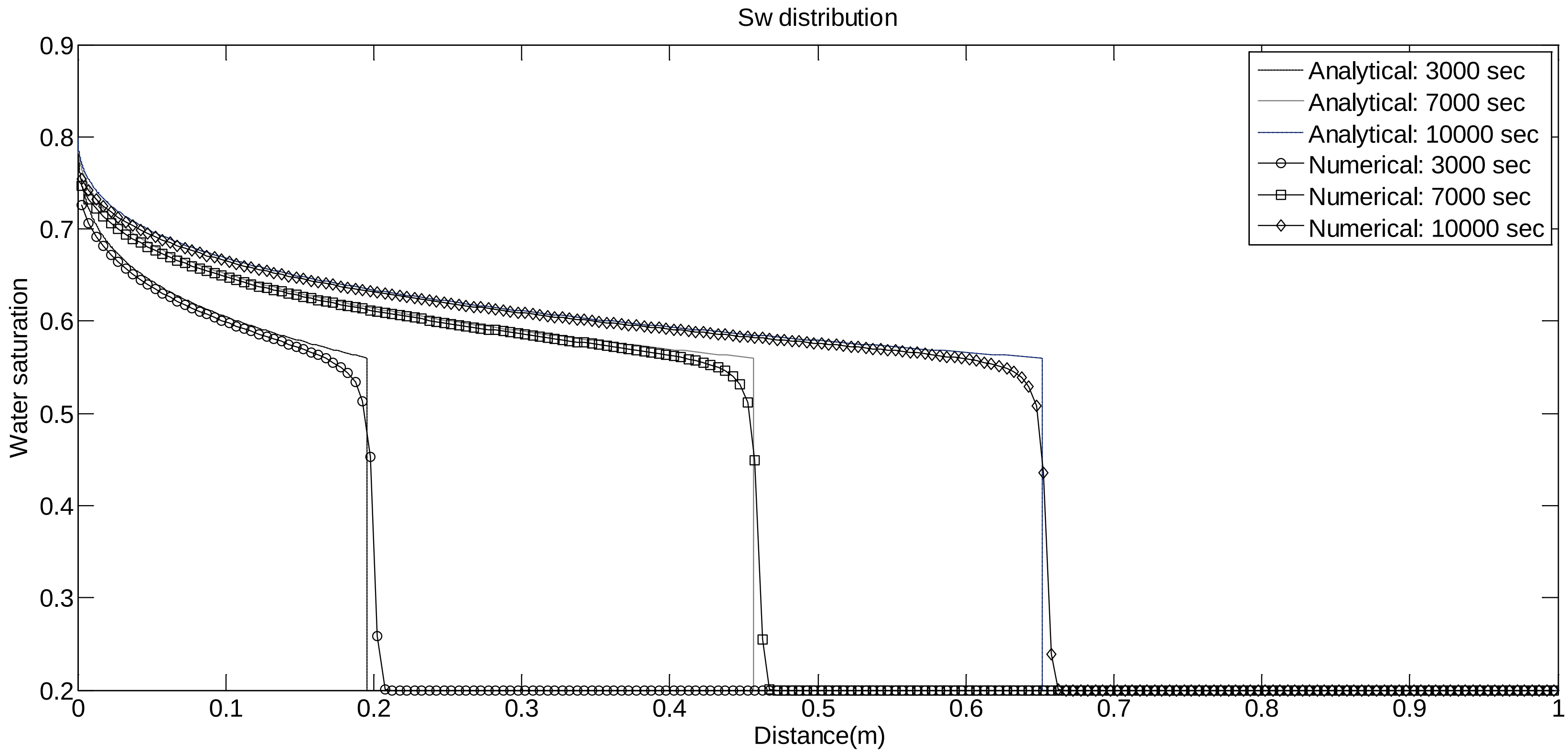
$$f_\alpha = \begin{cases} 0\,, \ \text{if} \quad f_\alpha < \left(f_\alpha\right)_c & \text{Darcy flow} \\ f_\alpha, \text{if} \quad f_\alpha > \left(f_\alpha\right)_c & \text{Forchheimer flow} \end{cases}$$

Validation: Buckley-Leverett problem with inertial effect

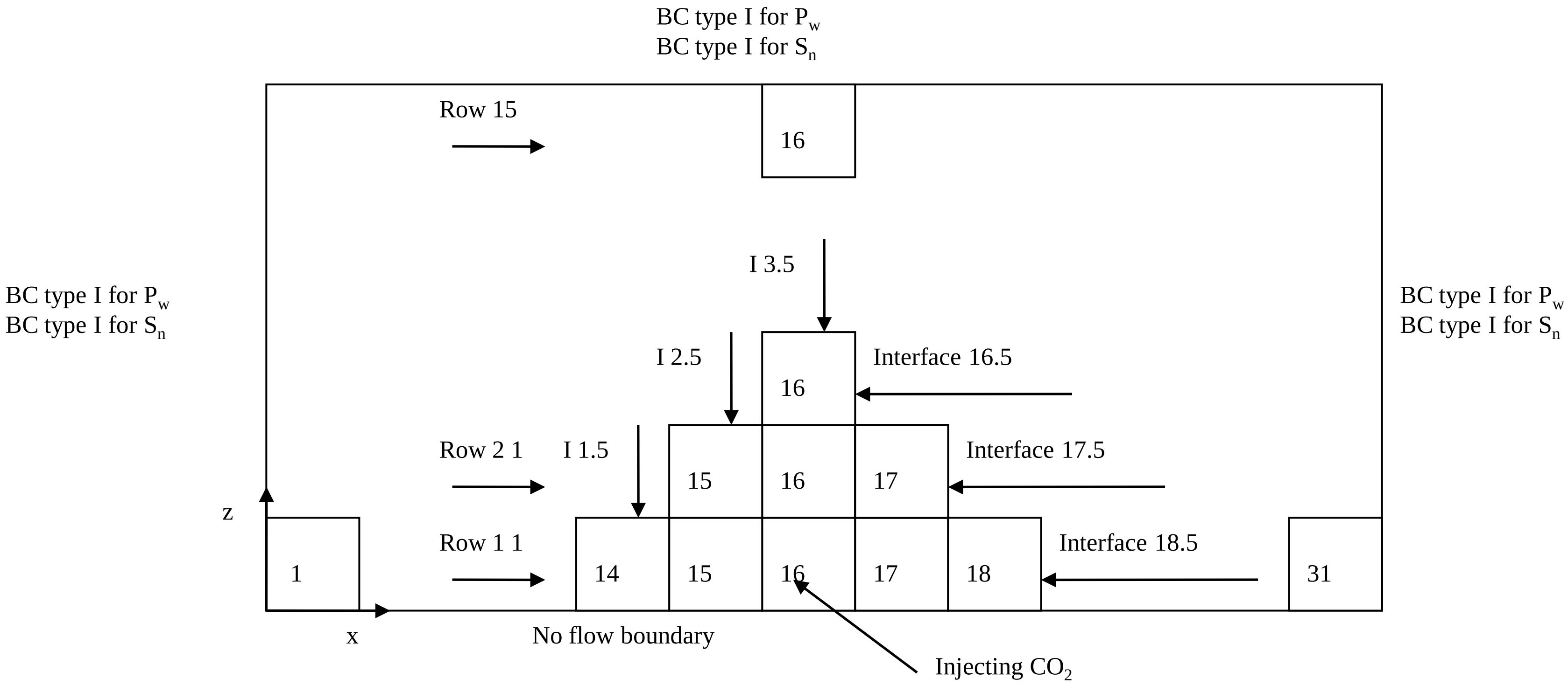
- Both fluids and the porous medium are incompressible;
- Capillary pressure gradient is negligible;
- Gravity effect is negligible;
- Semi-analytical solution with inertial effect (Wu, 2001; Ahmadi et al., 2010)



Comparison of saturation profiles



Application problem



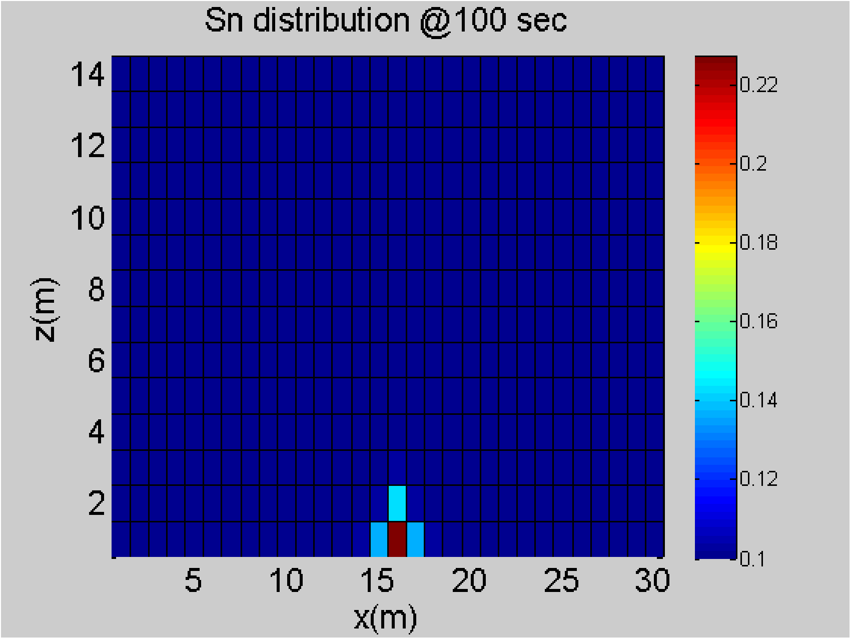
Properties of soil and fluids

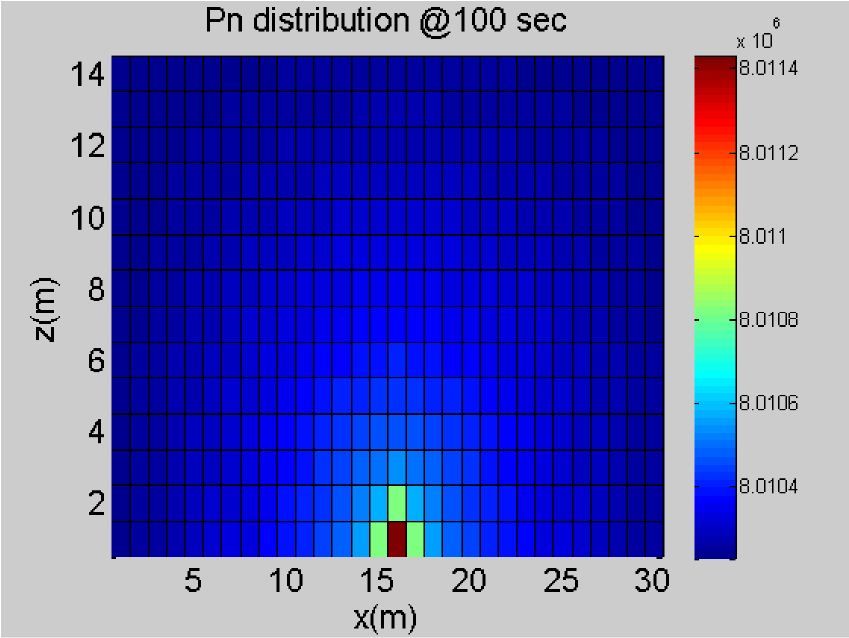
Properties	Values	Comment
Soil		
Soil intrinsic permeability porosity	3e-9 m^2 0.37	
Pore size distribution index	3.86	Brook-Corey
Water residual saturation	$S_{wr} = 0.35$	
Non-wetting phase (NWP) residual saturation	$S_{nr} = 0.05$	
Fluid		
Water density	994 kg/m^3	
NWP density	479 kg/m^3	
Water viscosity	7.43e-4 Pa s	
NWP viscosity	3.95 e-5 Pa s	

Modeling parameters

Properties	*Values*	**Comment**	
Boundary condition			
Water pressure at x=0.5 m Water pressure at x=31.5 m Water pressure at z=0.5 m Water pressure at z=15.5 m	P_w = 8 M Pa, BC Type I P_w = 8 M Pa, BC Type I No flow boundary P_w = 8 M Pa, BC Type I	Left boundary Right boundary Bottom boundary Top boundary	
CO_2 saturation at x=0.5 m CO_2 saturation at x=31.5 m CO_2 saturation at z=0.5 m CO_2 saturation at z=15.5 m CO_2 injecting rate @ (16,1)	S_n = 0.1, BC Type I S_n = 0.1, BC Type I No flow boundary S_n = 0.1, BC Type I $1*10^{-3}$ m^3/s	Per meter normal to the 2D domain	
Initial condition			
Water saturation NWP saturation Water pressure	S_w = 0.9 S_n = 0.1 P_w = 8 M Pa	Saturated with water initially	
Space discretization		**Time discretization**	
Domain size, Length Domain size, Depth Domain size, Width Space step size	L=31 m W=15 m 1 m dx =dz=1 m	Simulation time Time step size	T= 9000 s dt=1 s

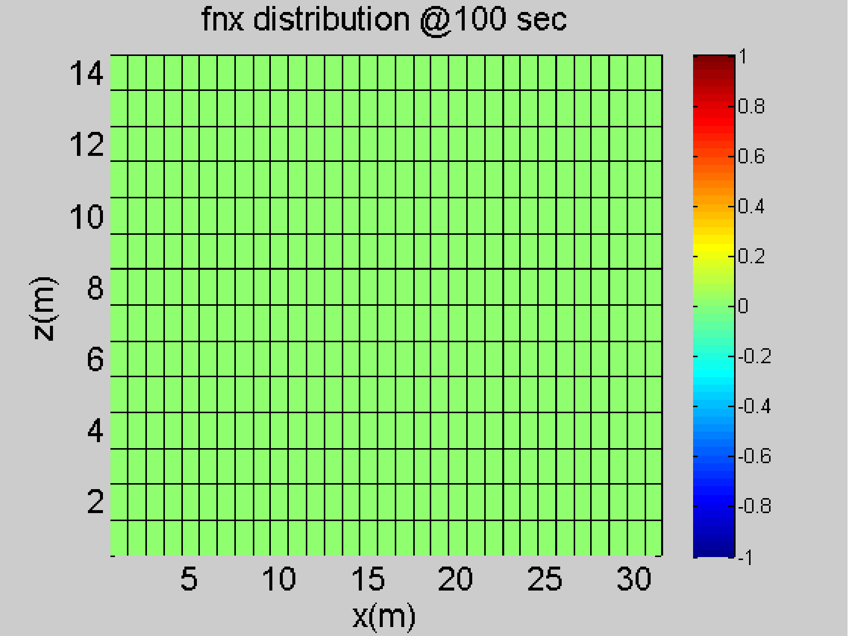
CO_2 saturation and pressure profiles

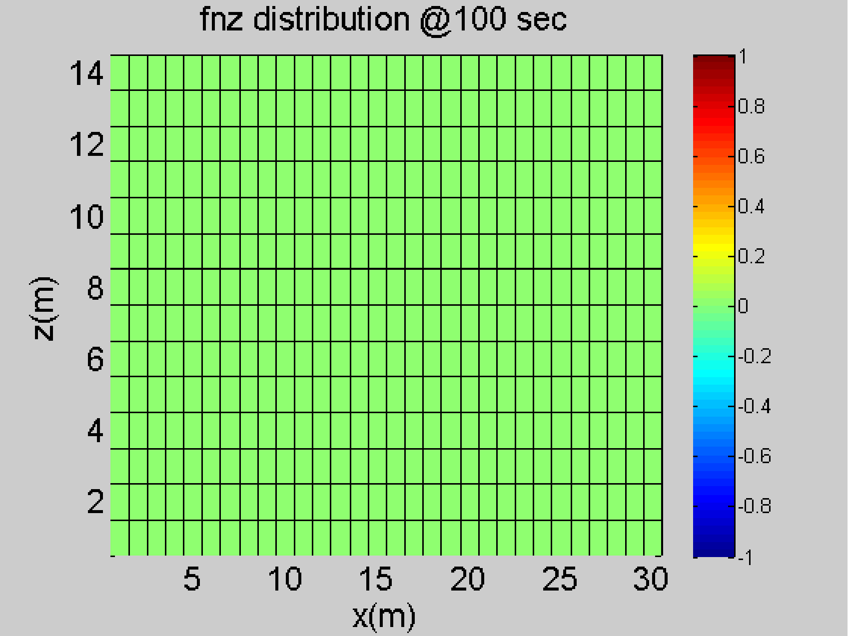




Darcy-Forchheimer flow

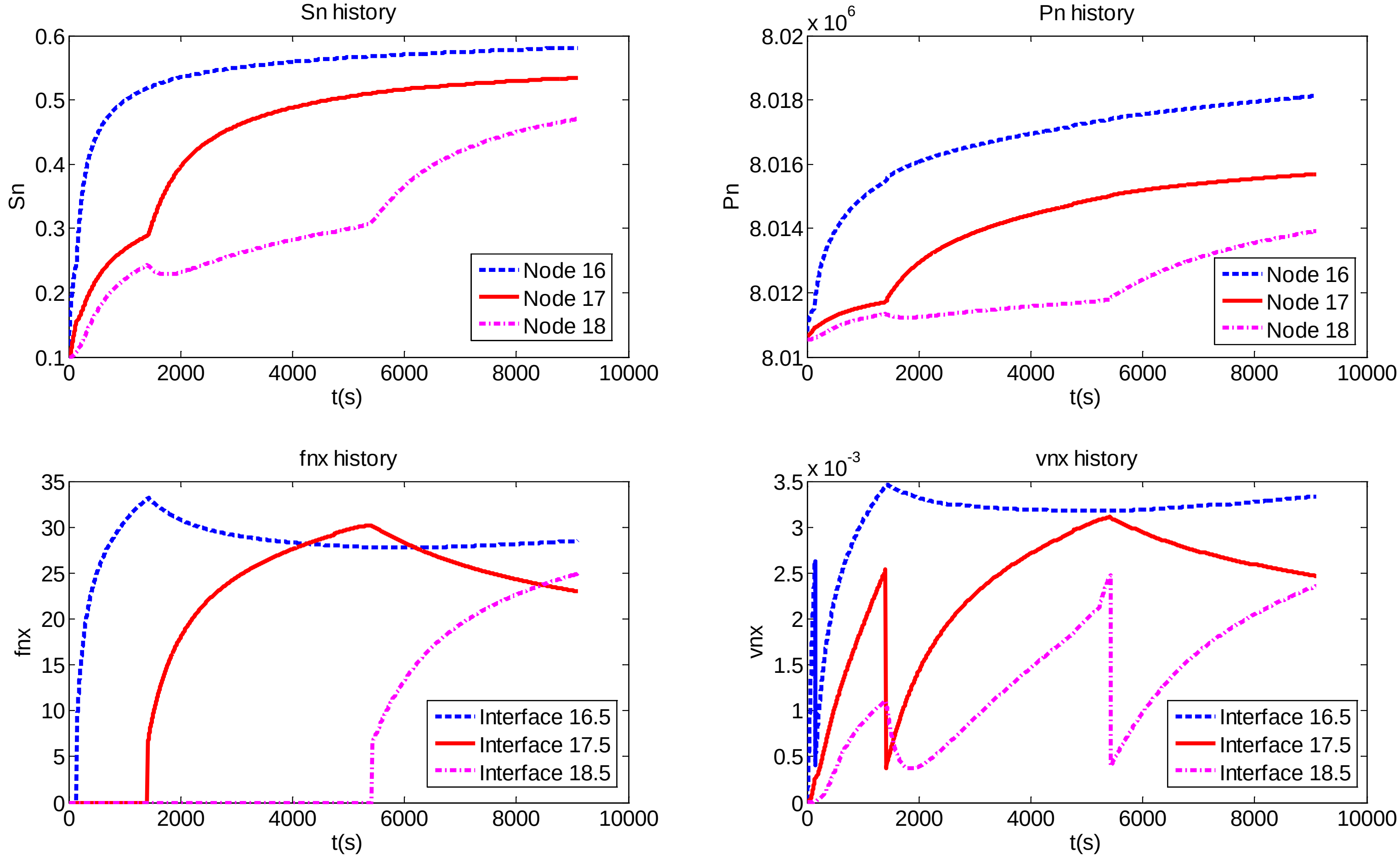
f_{nx} and f_{nz} profiles with time





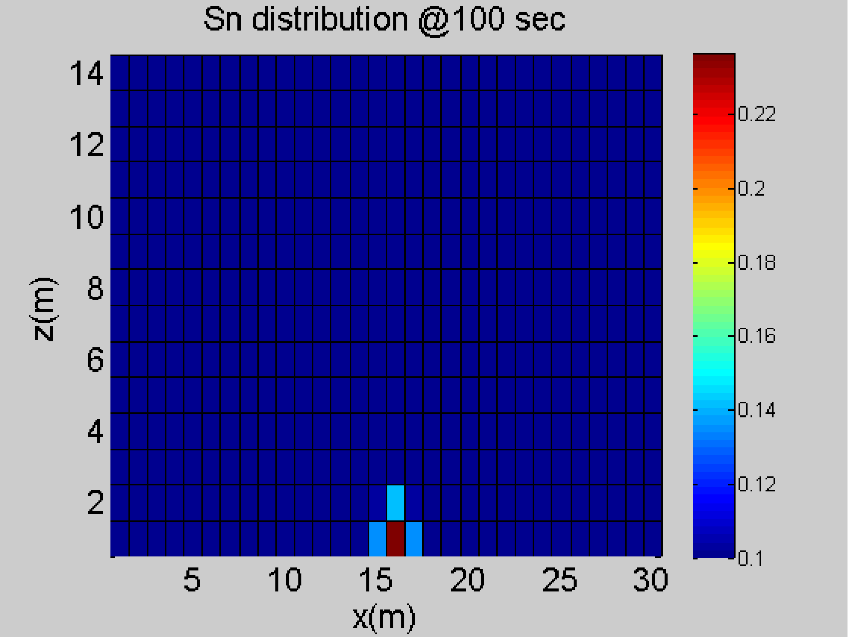
Darcy-Forchheimer flow

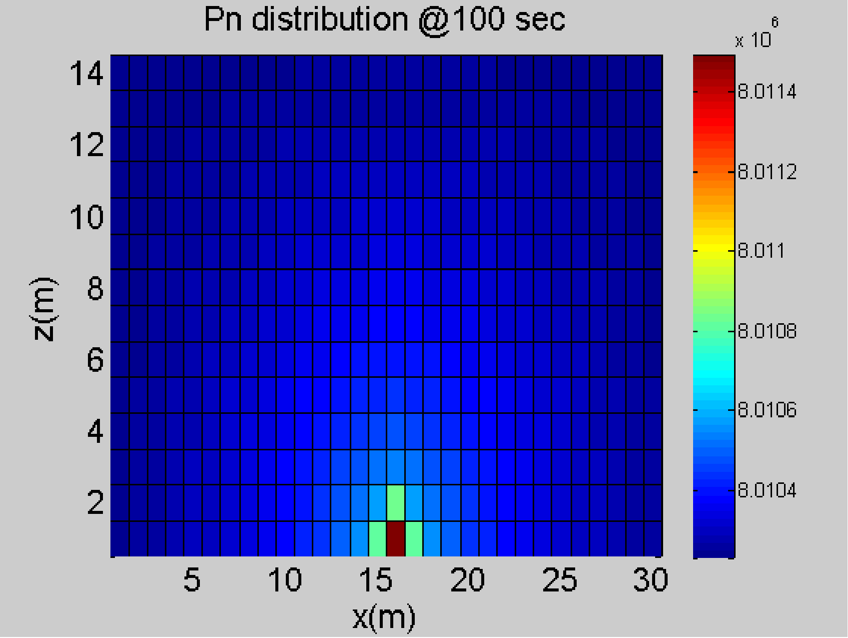
The evolution of important variables in the first row



Darcy-Forchheimer flow

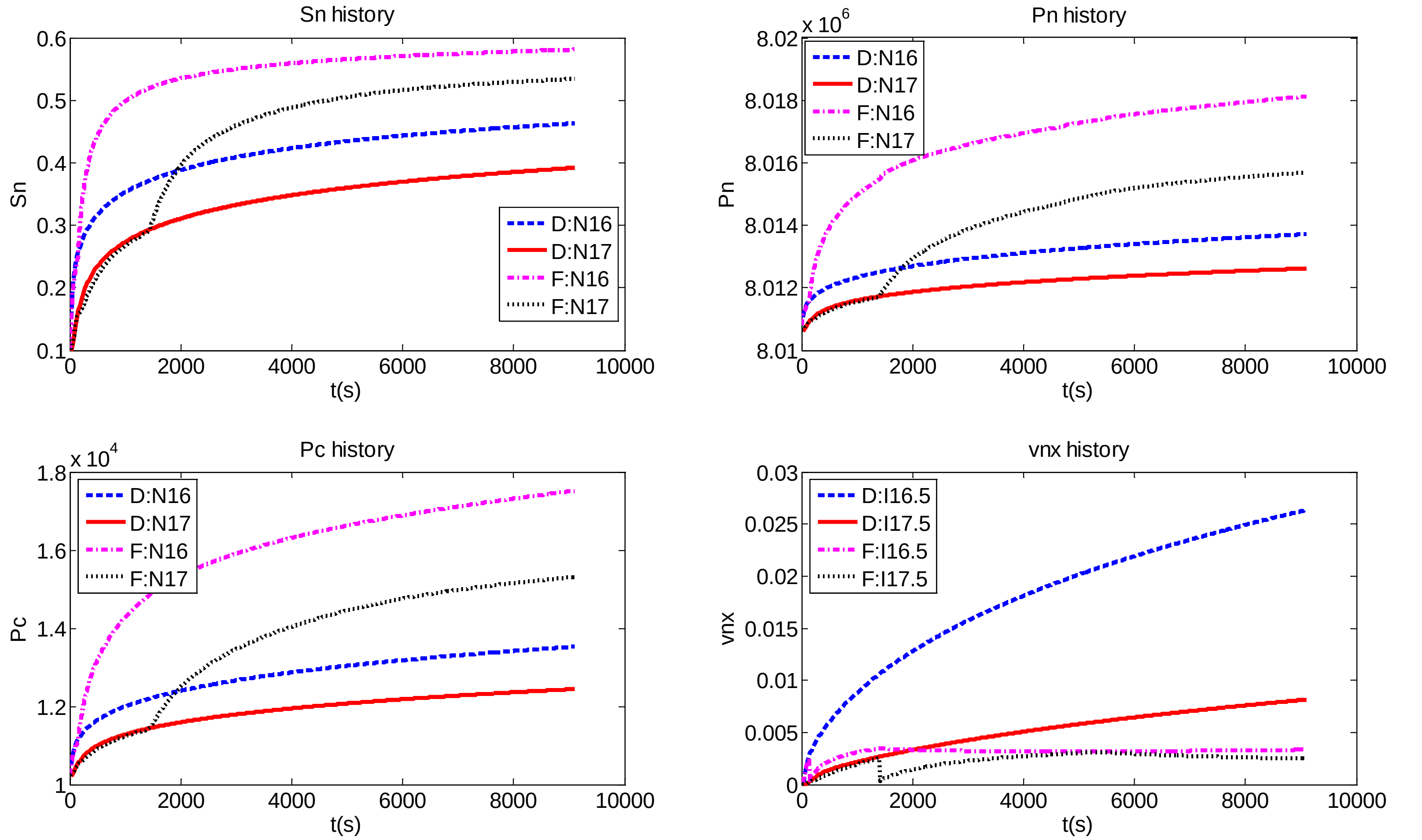
CO_2 saturation and pressure profiles



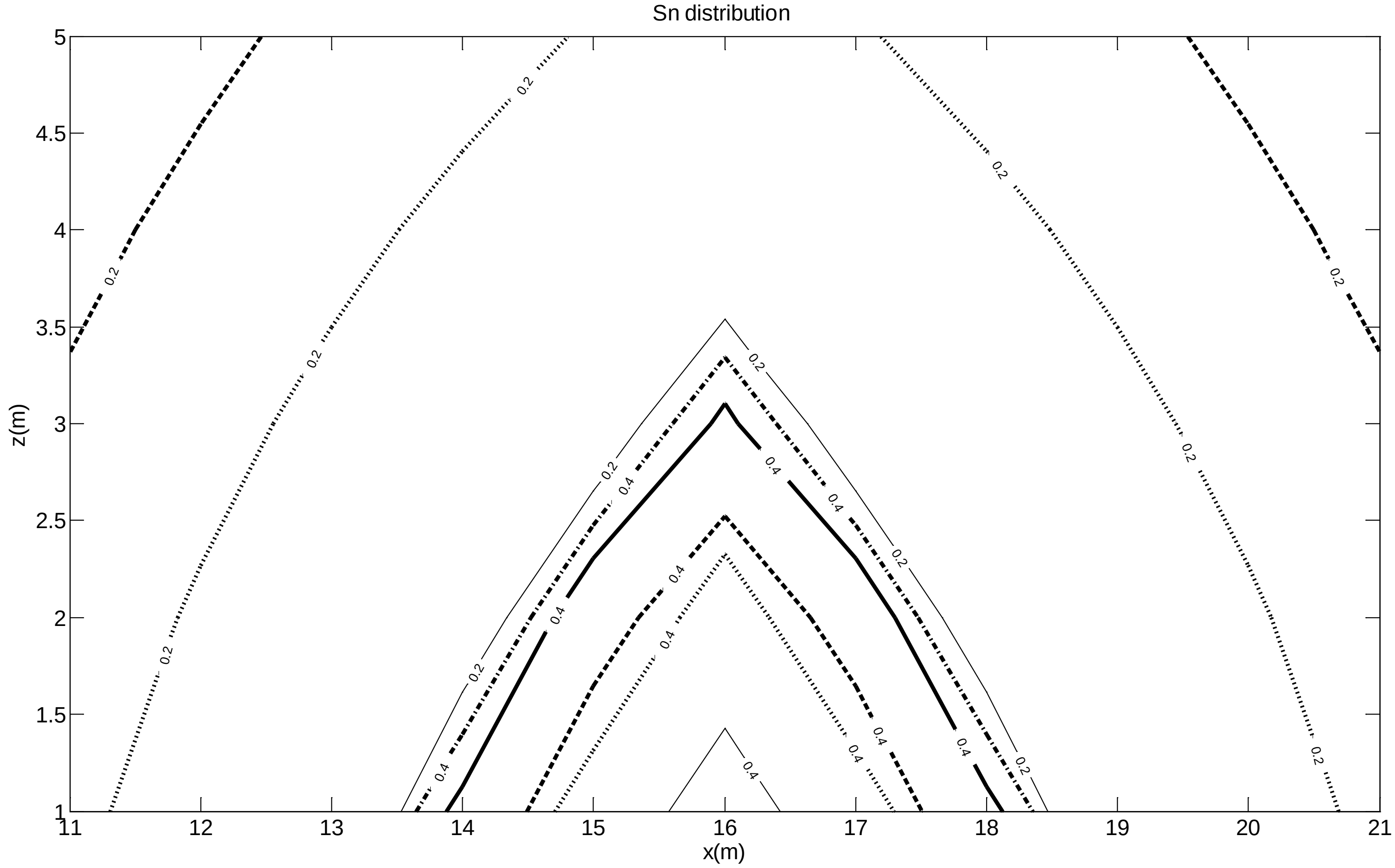


Darcy flow

Comparison of the evolution of important variables

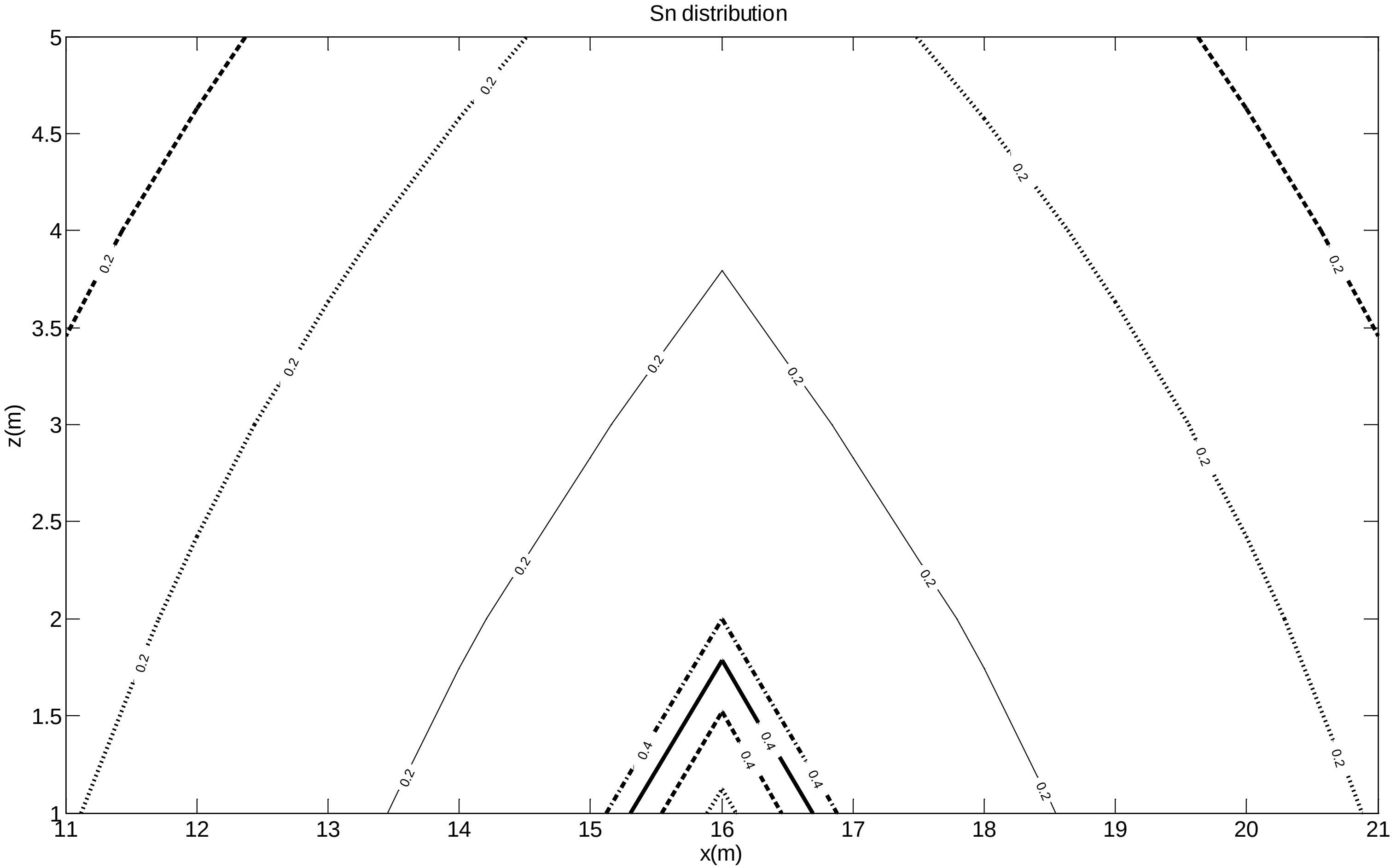


CO_2 saturation contour for Darcy-Forchheimer flow



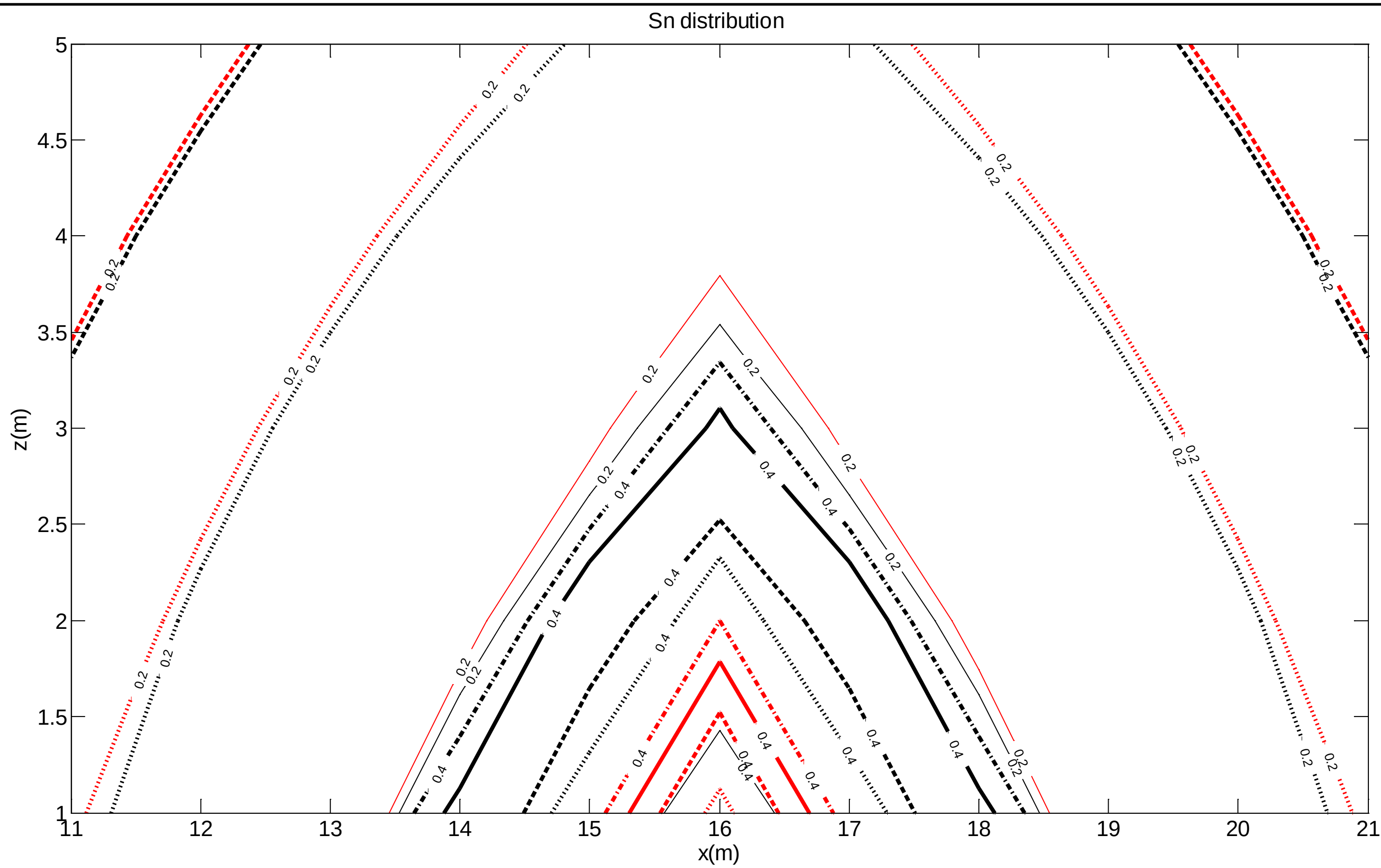
Time 1000 sec (solid line), 3000 sec (dot line), 5000 sec (dash line), 7000 sec (bold solid line), 9000 sec (dash dot line)

CO_2 saturation contour for Darcy flow



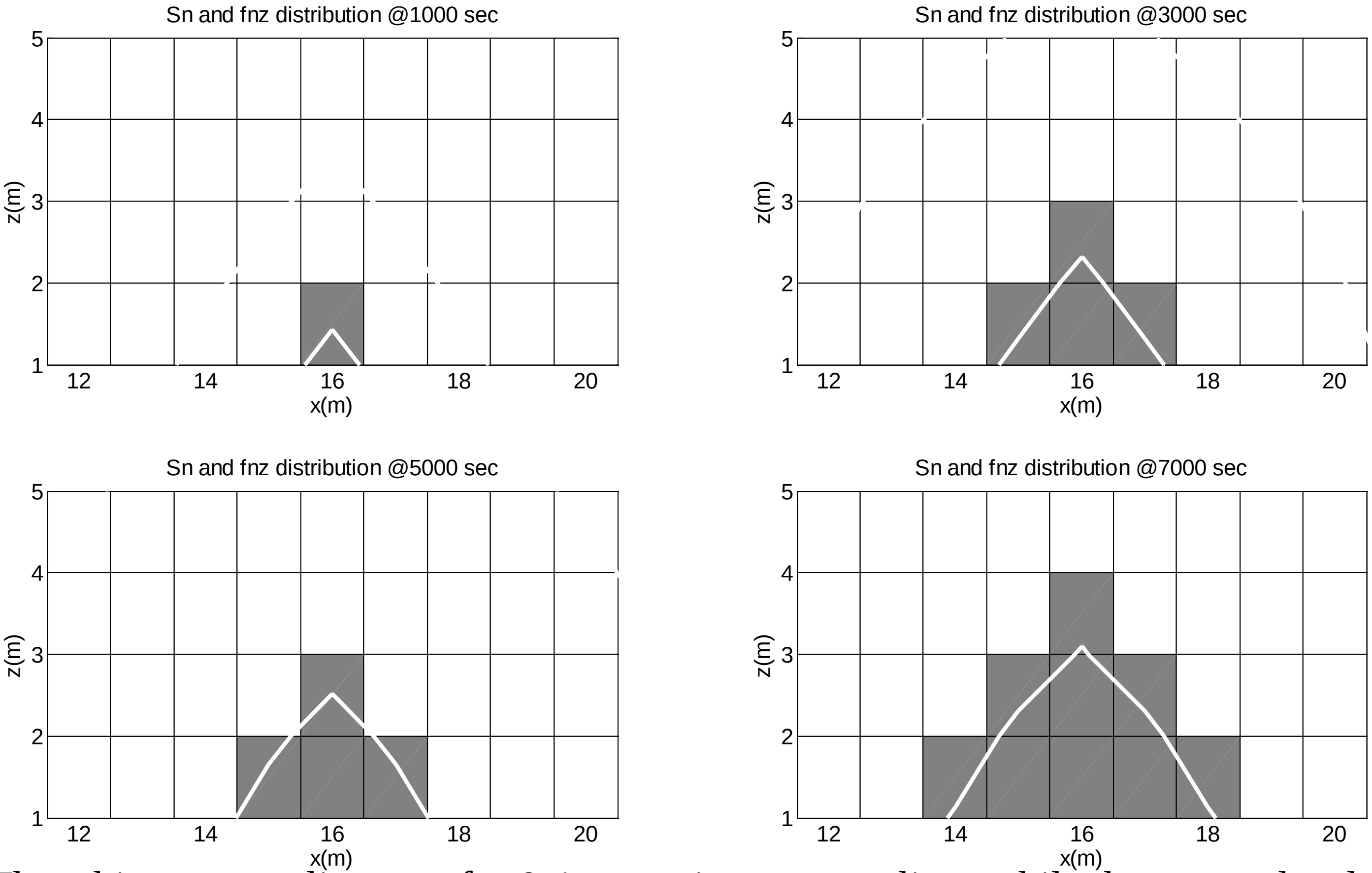
Time 1000 sec (solid line), 3000 sec (dot line), 5000 sec (dash line), 7000 sec (bold solid line), 9000 sec (dash dot line)

Overlapping them together



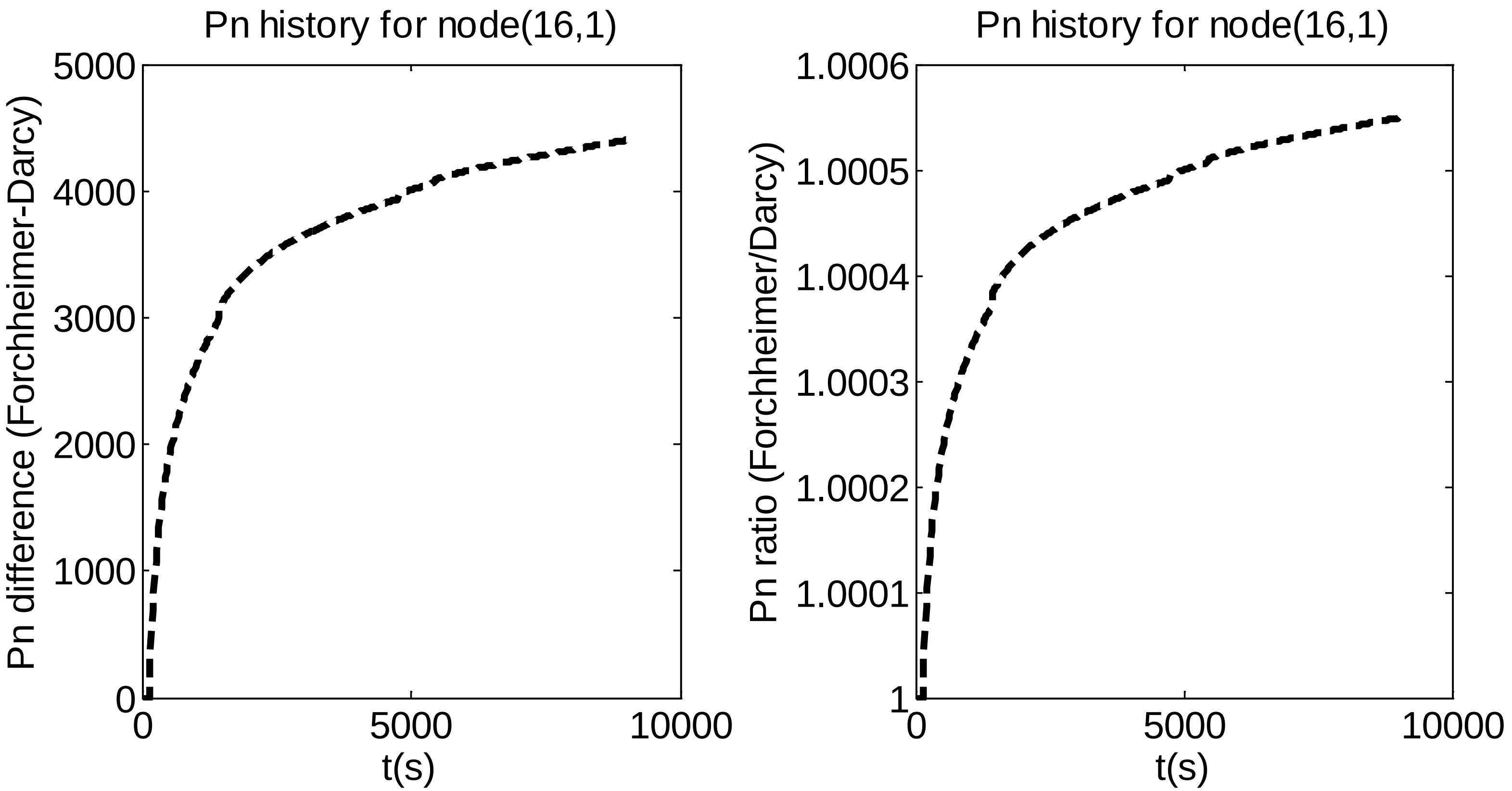
Time 1000 sec (solid line), 3000 sec (dot line), 5000 sec (dash line), 7000 sec (bold solid line), 9000 sec (dash dot line); Darcy in red; Darcy-Forchheimer in black

Match S_n with Forchheimer flow regime



The white contour lines are for 0.4 saturation contour lines while the rectangles demonstrate whether a node is of Forchheimer (grey rectangle) or Darcy (white rectangle) flow

Comparison of CO_2 pressure between Forchheimer-Darcy and Darcy flow



Higher displacement efficiency

- In the Forchheimer regime for Darcy-Firchheimer flow, the total CO_2 saturation is 4.5916 for the nine nodes at 9100 sec.

- For Darcy flow, the total CO_2 saturation is 3.4072 for the same nine nodes at 9100 sec.

- The displacement is 34.76% higher for Forchheimer flow than Darcy flow at 9100 sec.

Implications

- Important to incorporate Forchheimer effect into the numerical simulation of multiphase flow
- Crucial to determine the critical Forchheimer number and to decide the extent to which Forchheimer effect can influence the transport of CO_2 in deep saline aquifers.
- The higher displacement efficiency by CO_2 is good news for CO_2 sequestration into deep saline aquifers.
- The higher injection pressure required in Forchheimer flow is bad news for CO_2 sequestration.

Summary & conclusions

- Darcy flow is a special case of a generalized Darcy-Forchheimer flow;
- Since both the Forchheimer coefficient and number are functions of saturation, there is a critical Forchheimer number for transition for a specific saturation for each phase in multiphase flow;
- The good agreement between the numerical solution and the semi-analytical solution validates the numerical tool developed in this study

Summary & conclusions

- The Forchheimer flow can improve the displacement efficiency and can increase the storage capacity for the same injection rate and volume of site.

- The higher injection pressure required in Forchheimer flow is bad news for CO_2 sequestration because the pressure will continue to increase and might even exceed the litho-static stress and the risk for fracturing the porous media would increase.

Thank you!