Water Sensor Placement in Water Distribution Systems

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Measures of performance:

- ✓ **Time of detection (Z_1)**
- ✓ **Population effected (Z_2)**
- ✓ **Water volume contaminated (Z_3)**
- ✓ **Reliability (Z_4)**

NOTE: Measures are assumed to be of equal weight.

Cases:

	Injection Duration	Response Delay	Number of Injection Events*
Base Case A	**2 h**	**0 h**	**1**
Derivative Case B	**10 h**	**0 h**	**1**
Derivative Case C	**2 h**	**3 h**	**1**
Derivative Case D	**2 h**	**0 h**	**2**

Attack scenarios are generated in a MC sense.

* All nodes have initially equal probabilities of attack. Multiple nodes attacked are attacked simultaneously.

Performance metrics:

	Performance Metrics			
"Importance" Variables	**Detection Time**	**Population Affected**	**Contaminated Demand**	**Detection Likelihood**
Contaminant Concentration		X	X	X
Hydraulic Demand		X	X	
Time after contamination event	X	X	X	
Contamination occurrence				X

"Importance" of a node variable:

- **To represent the goals of the 4 performance metrics**

Goals:

- **Find minimum number of water sensors necessary for a specified reliability level for the system.**
- **Find optimal water sensor placement for these sensors.**
- **Overall, the network designed should have high reliability, minimum time of detection, minimum affected population and low volume of contaminated water consumed.**

Mathematical formulation:

- Objective function:

$$F = \underset{X}{\text{minimize}} \left\{ \left(1 - r\left(X\right)\right) \times E_s \left[\sum_{i=1}^{N} \sum_{t=t_s^{in}}^{t_s^d(X)} \left(t - t_s^{in} + 1\right) V_{is}(t) \right] \right\}$$

X: decision variable vector, $X = [x_1, x_2, \ldots, x_{Nd}]$, $x_i = \{0, 1\}$

$r(X)$: reliability of the system

V_{is} : volume of consumed contaminated water

t_s^{in}: index of injection time

$t_s^d(X)$: the time of detection for X

Mathematical formulation:

- Objective function:

$$F = \underset{X}{\text{minimize}} \left\{ \frac{\left(1 - r\left(X\right)\right)}{N_s} \sum_{s=1}^{N_s} \left[\sum_{i=1}^{N} \sum_{t=t_s^{in}}^{t_s^d(X)} \left(t - t_s^{in} + 1\right) V_{is}(t) \right] \right\}$$

N: number of total junctions

N_s: number of the contamination events

N_d: number of candidate sensors

This choice implies:

- Volume contaminated and population affected?

  Minimize objective value:

  $\Rightarrow$ reduce the volume of contaminated water

  $\Rightarrow$ reduce the population affected.

- Detection time? $\Rightarrow \left\{ \begin{array}{c} reduce\ summation\ terms \\ reduce\ coefficicant\ (t - t_s^{in} + 1) \end{array} \right\}$

  $\Rightarrow reduce\ objective\ value$

- High reliability? $\Rightarrow$ reduce {1-r(X)} $\Rightarrow$ reduce objective value

Calculation of parameters:

- Time of detection:

$$t_s^d(X) = \min_j \left\{ t_{js} \right\}$$

j: index of sensors in solution *X*

t_{js}: time of detection at sensor j for event s

Calculation of parameters:

- Volume of contaminated water:

$$V_{is}(t) = \begin{cases} 0 & if \ \ C_{is}(t) < C_{\min} \\ q_i(t)\Delta t & if \ \ C_{is}(t) \geq C_{\min} \end{cases}$$

C_{min}: threshold hazard concentration

C_{is}: concentration at junction i for event s

$q_i(t)$: actual water demand at junction i at time step t

Δt: time step interval

Calculation of parameters:

- Reliability:

$$r = \frac{1}{N_s} \sum_{s=1}^{N_s} d_s(X)$$

$$d_s = \begin{cases} 1 & \text{contaminant scenario s is detected} \\ 0 & otherwise \end{cases}$$

Constraints:

- Probability of detected events:

$$\sum_{i=1}^{N_d} x_i = M(r_c)$$

$$r = prob\{\text{detected events} / M \leq m\} \geq r_c$$

r_c: specified reliability of system

$M(r_c)$: number of sensors needed for r_c

m: a specified number of sensors

Iterative steps to determine N_d

Decomposition - Coordination

Find optimal placement of water sensors for specified number of water sensors using IGA

r_{opt} (X) M^l

Find minimum number of water Sensors using interpolation

Subdomain? (IGA)

- **Selection of the subdomain:** a subset of all junctions selected by roulette wheel based on average water demand or average time of detection

- **Composition of chromosomes:** all x_i of the subdomain consist of a chromosome.

- **Initialization of population:** the initial population of sensors is randomly generated with uniform distribution within the subdomain.

IGA

- Fitness calculation:

$$Fitness_k = f_{\max} - f_k$$

- Selection of mating pool: the mating pool is selected from current population using roulette wheel method

- Generation of new population: the new population is generated using genetic operators including crossover, new member generation, mutation and elitism

IGA

- Evolution: the population generated by the operators above replaces the current population to produce a new generation.

- Update of the subdomain: a new subdomain is generated to replace the current one. The new subdomain must include those junctions of the best solution of the prior evolution. Process is repeated until all junctions are used at least once.

Coordination:

- Criterion for determining the number of sensors

$$\begin{cases} |r - r_c| \le \varepsilon & \textit{stop calculation} \\ r < r_c & \textit{increase number of sensors} \\ r > r_c & \textit{decrease number of sensors} \end{cases}$$

Coordination:

- Determination of number of sensors

$$M^{(l+1)} = M^{(l)} + \Delta M$$

l : index of iterations

ΔM: incremental number of sensors

Coordination:

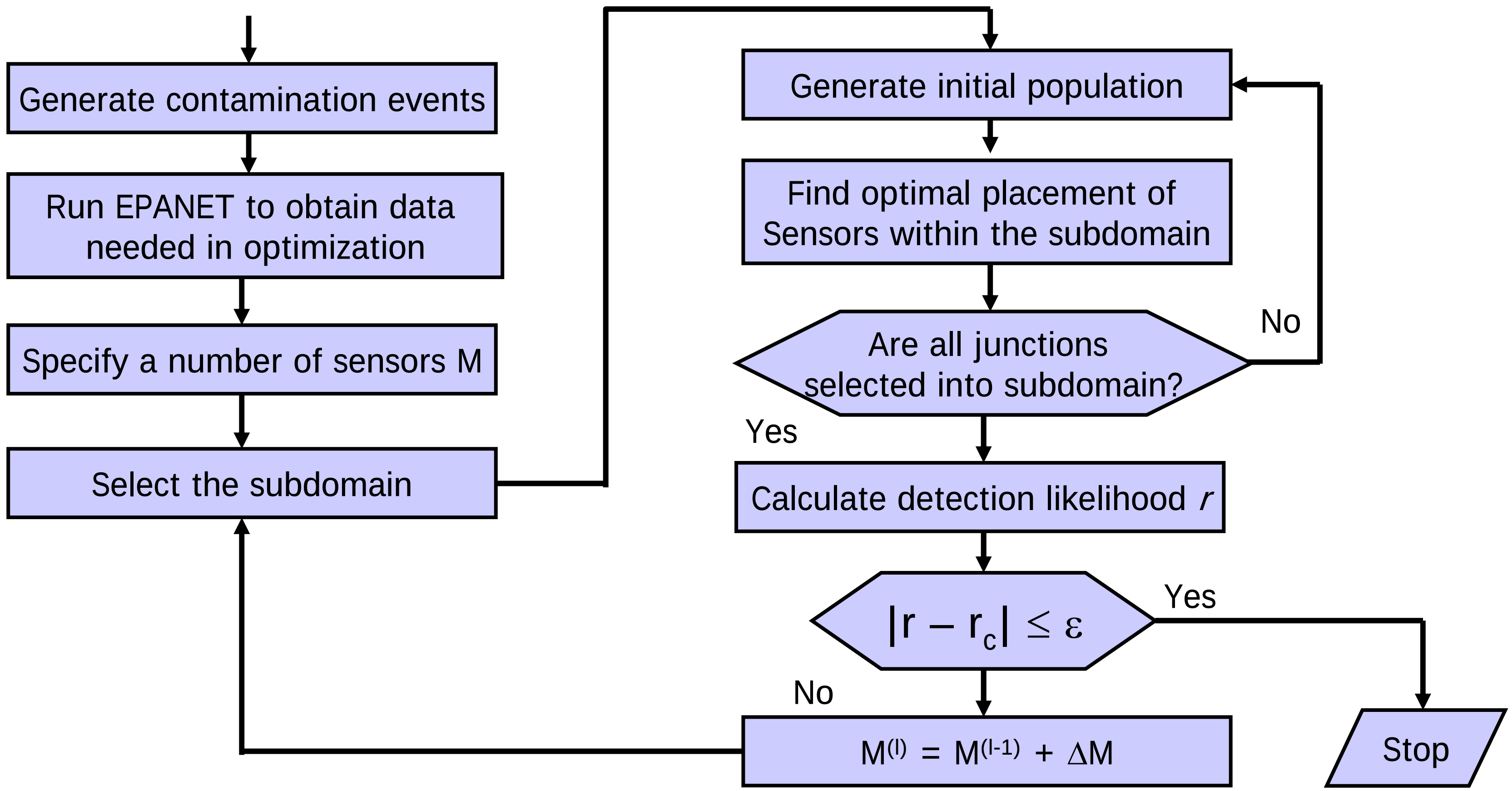
- For normal case that the reliability is monotonically increasing function of number of sensors for $l > 0$:

$$\Delta M = \left[\frac{r_c - r^{(l)}}{r^{(l)} - r^{(l-1)}} \left(M^{(l)} - M^{(l-1)} \right) \right]$$

- For case with l = 0 or abnormal cases:

$$\Delta M = \begin{cases} 1 & r^{(l)} < r_c \\ -1 & r^{(l)} > r_c \end{cases}$$

IGA flowchart:

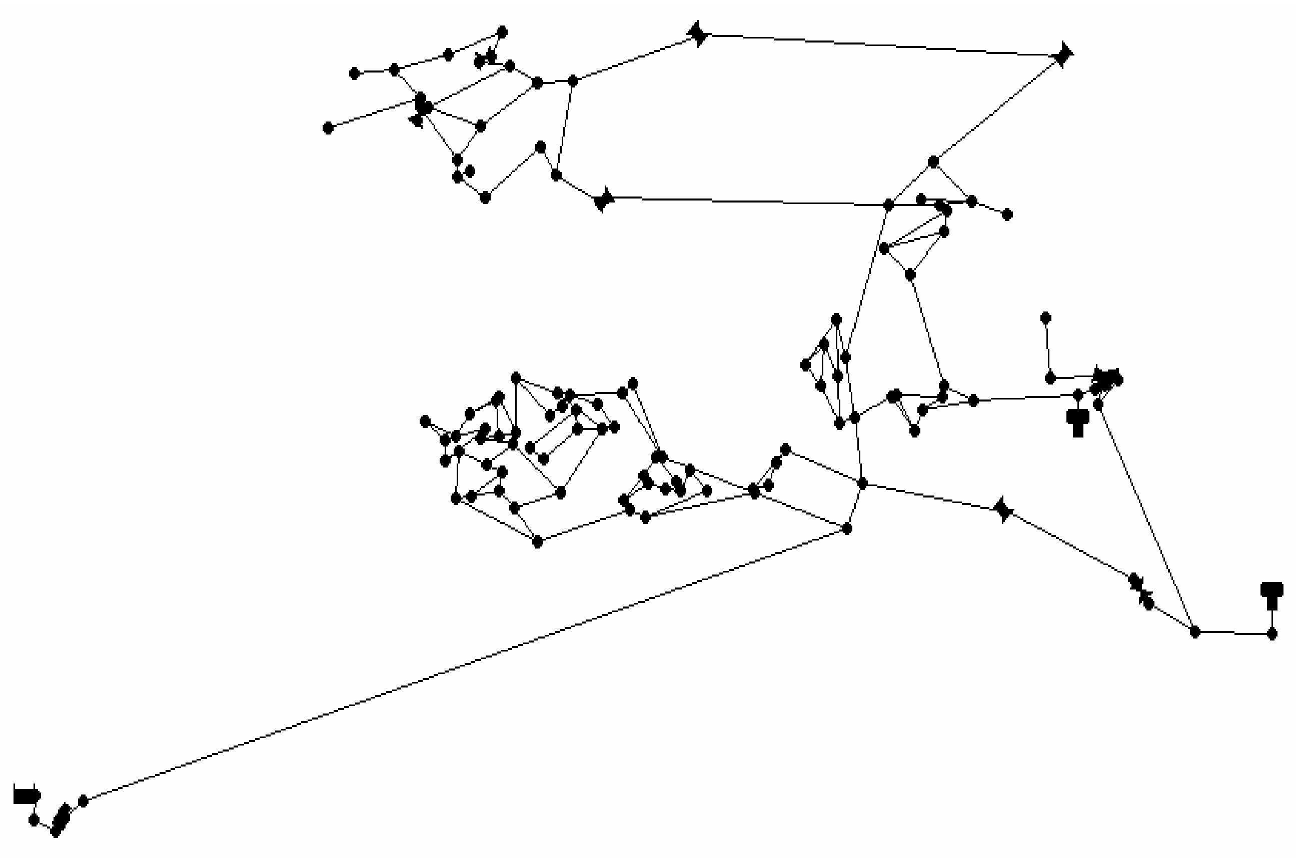


GA parameters:

Parameters	**Value**
Population size	50
Crossover ratio	0.8
New member generation ratio	0.2
Elitism ratio	A best member
Mutation ratio	0.2
Maximum generation for each subdomain	30
Number of scenarios in optimization	2,580
Number of scenarios in verification	2,580

Applications: WDS -1

- 126 Junctions
- 168 Pipes
- 1 Reservoir
- 2 Tanks
- 2 Pumps
- 8 Valves



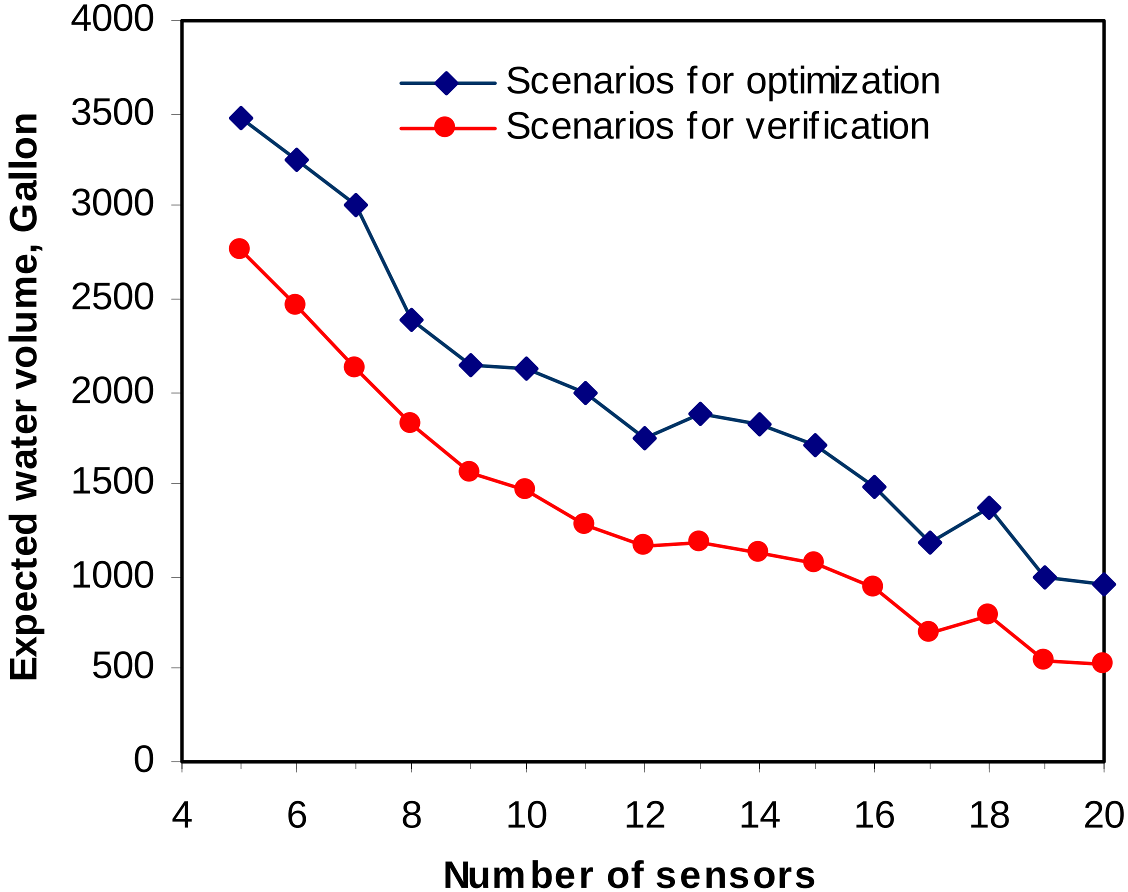
WDS-1: Design stage

# of Sensors	Junction ID	Z_1 (minutes)	Z_2	Z_3 (Gal)	Z_4 (%)
5	J17, J31, J81, J98, J102	409.05	200.56	3482.09	66.51
6	J20, J68, J84, J98, J102, J118	339.23	248.97	3254.67	68.84
7	J20, J68, J82, J84, J98, J103, J118	324.95	235.25	2996.49	71.94
8	J17, J23, J46, J68, J83, J101, J103, J118	384.10	183.05	2377.35	76.43
9	J17, J23, J39, J46, J68, J83, J101, J103, J118	353.32	172.99	2147.19	76.43
10	J17, J23, J39, J45, J68, J83, J101, J102, J118, J122	360.18	156.86	2115.58	78.76
18	J4, J11, J17, J21, J27, J31, J35, J46, J68, J75, J79, J82, J83, J96, J100, J118, J122, J126	310.91	125.58	1372.86	84.85
19	J4, J11, J17, J21, J27, J31, J34, J46, J68, J75, J82, J83, J95, J100, J102, J118, J122, J126	320.07	119.98	1004.37	85.50
20	J4, J11, J17, J21, J27, J31, J34, J35, J46, J68, J75, J79, J82, J83, J98, J100, J102, J118, J122, J126	287.22	122.23	956.08	85.50

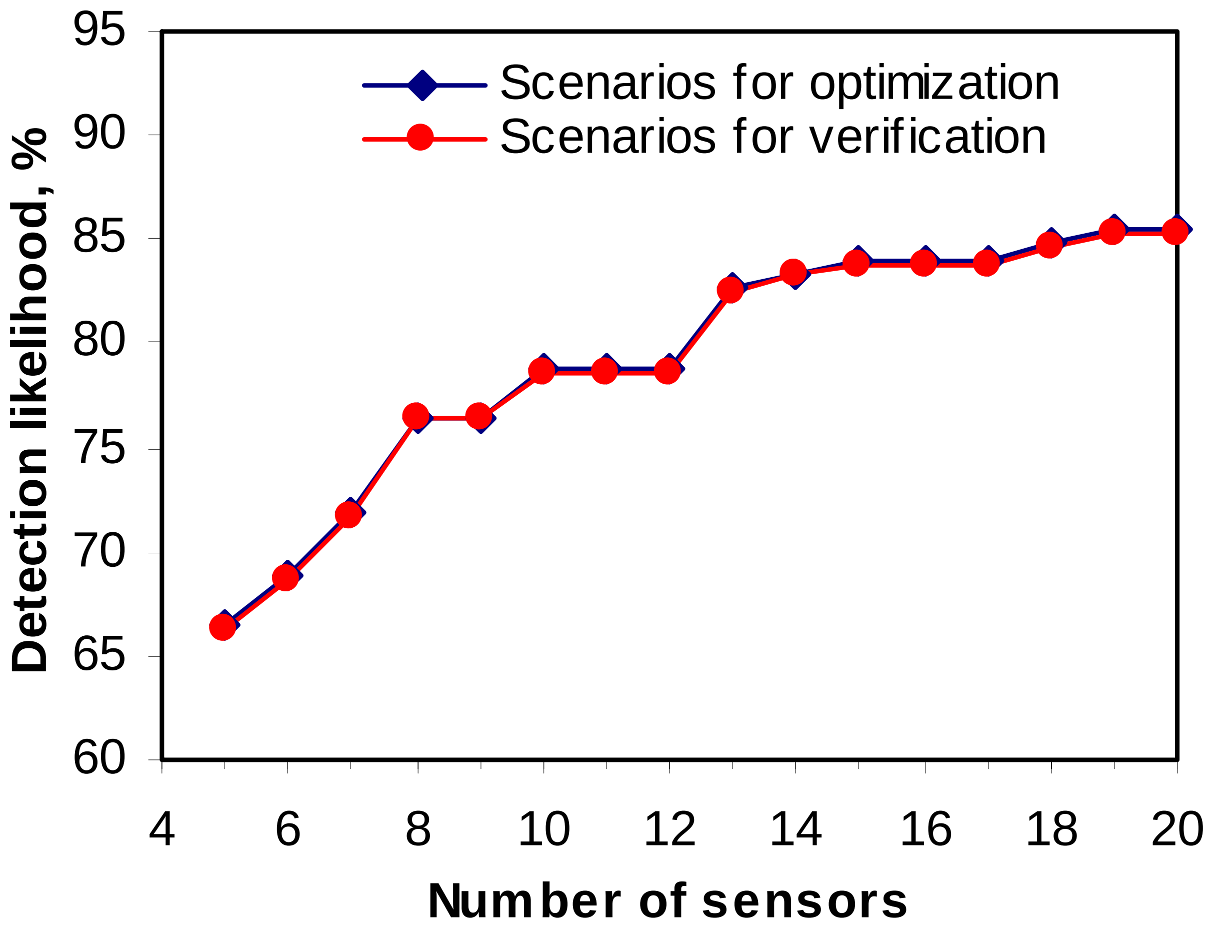
WDS-1: Verification stage

# of Sensors	Z_1 (minutes)	Z_2	Z_3 (Gal)	Z_4 (%)
5	632.77	158.87	2758.23	66.32
6	515.86	188.29	2461.44	68.64
7	474.92	175.05	2112.90	71.74
8	531.77	132.11	1814.91	76.36
9	487.15	125.37	1556.04	76.36
10	482.30	122.50	1465.37	78.68
18	350.30	82.86	792.27	84.65
19	366.49	76.67	550.11	85.31
20	330.42	78.21	525.72	85.31

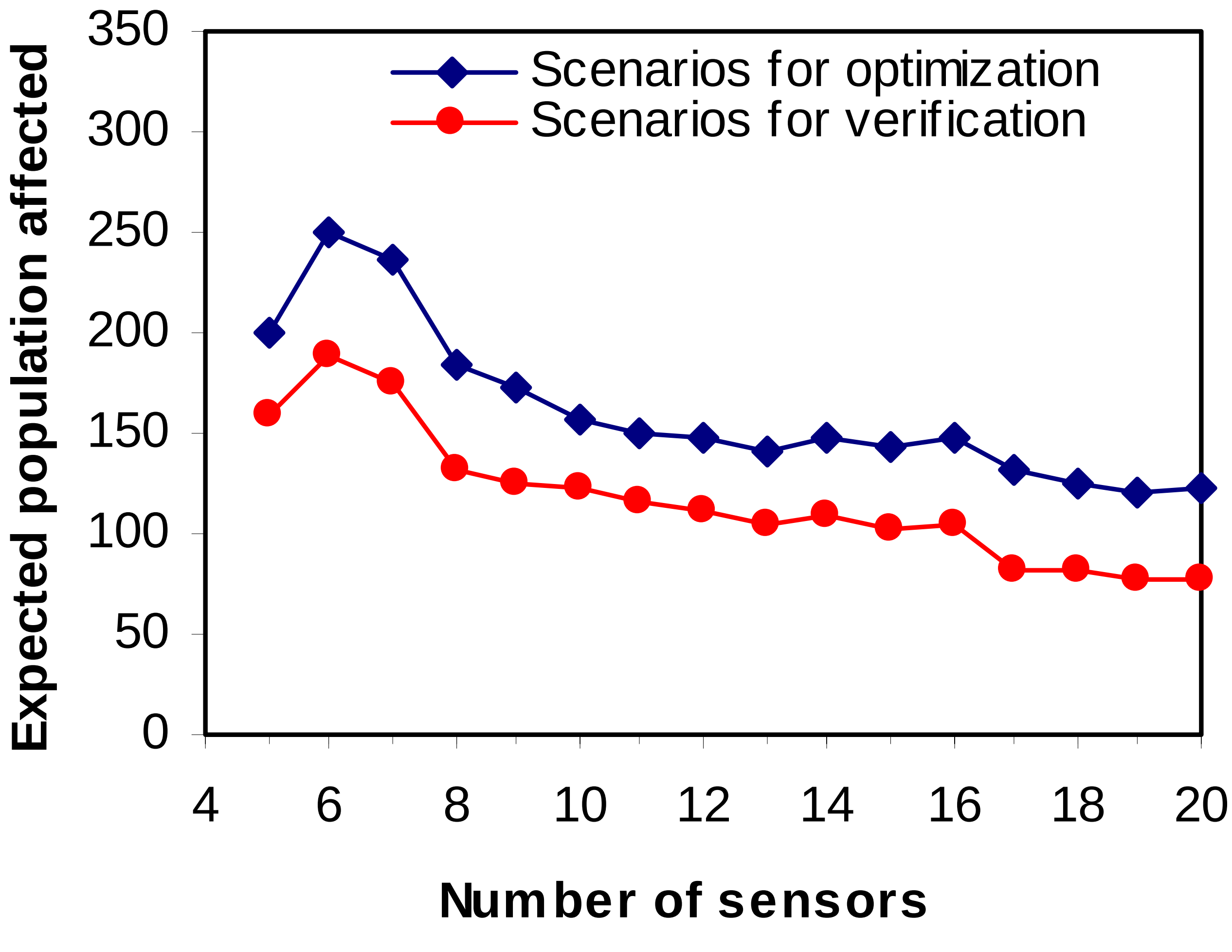
WDS-1: Volume vs # of sensors



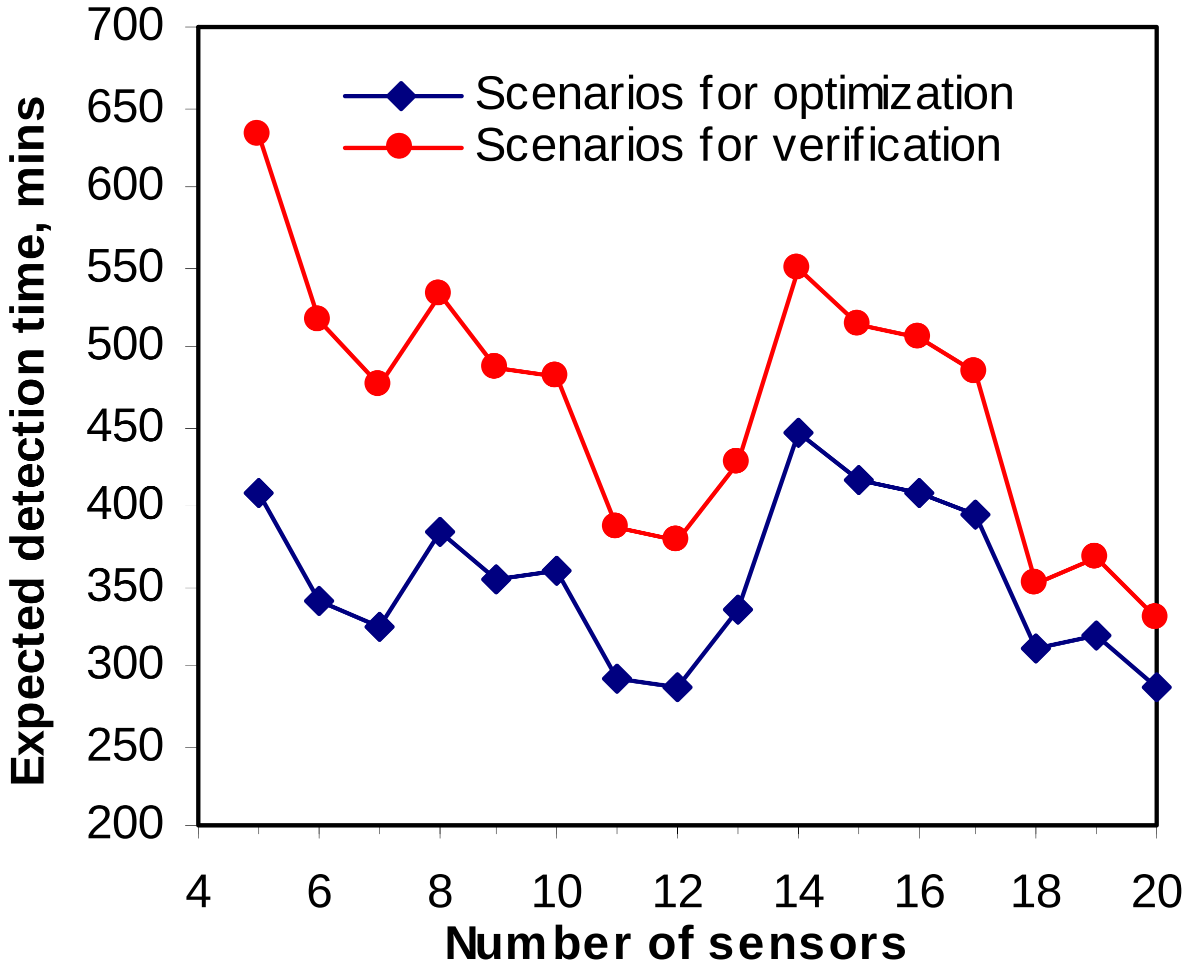
WDS-1: Reliability vs # of sensors



WDS-1: Population vs # of sensors



WDS-1: Detection time vs # of sensors



Mathematical formulation:

- Objective function:

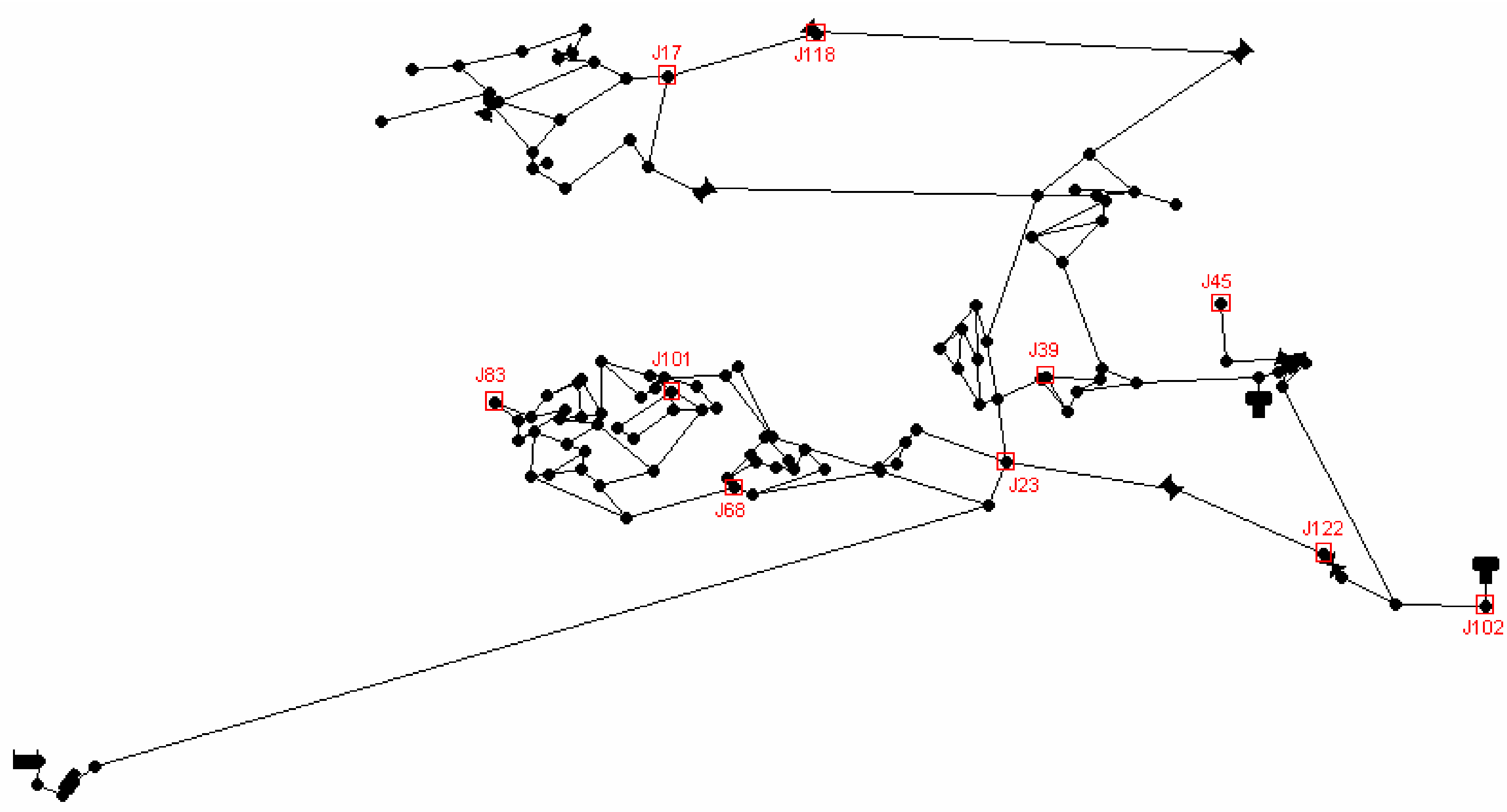
$$F = \underset{X}{\text{minimize}} \left\{ \frac{\left(1 - r(X)\right)}{N_s} \sum_{s=1}^{N_s} \left[\sum_{i=1}^{N} \sum_{t=t_s^{in}}^{t_s^d(X)} \left(\left(t - t_s^{in} \right) + 1 \right) V_{is}(t) \right] \right\}$$

N: number of total junctions

N_s: number of the contamination events

N_d: number of candidate sensors

WDS-1: Optimal locations

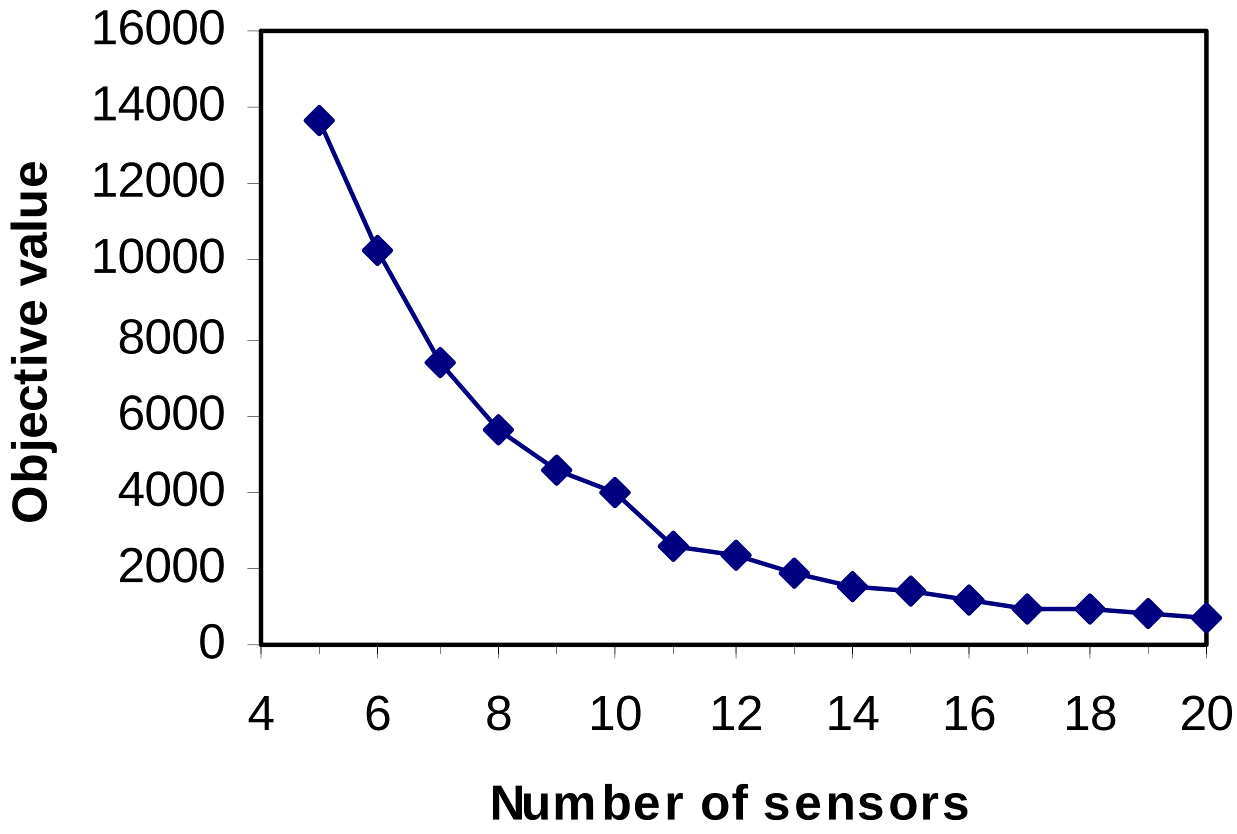


Impact of objective function:

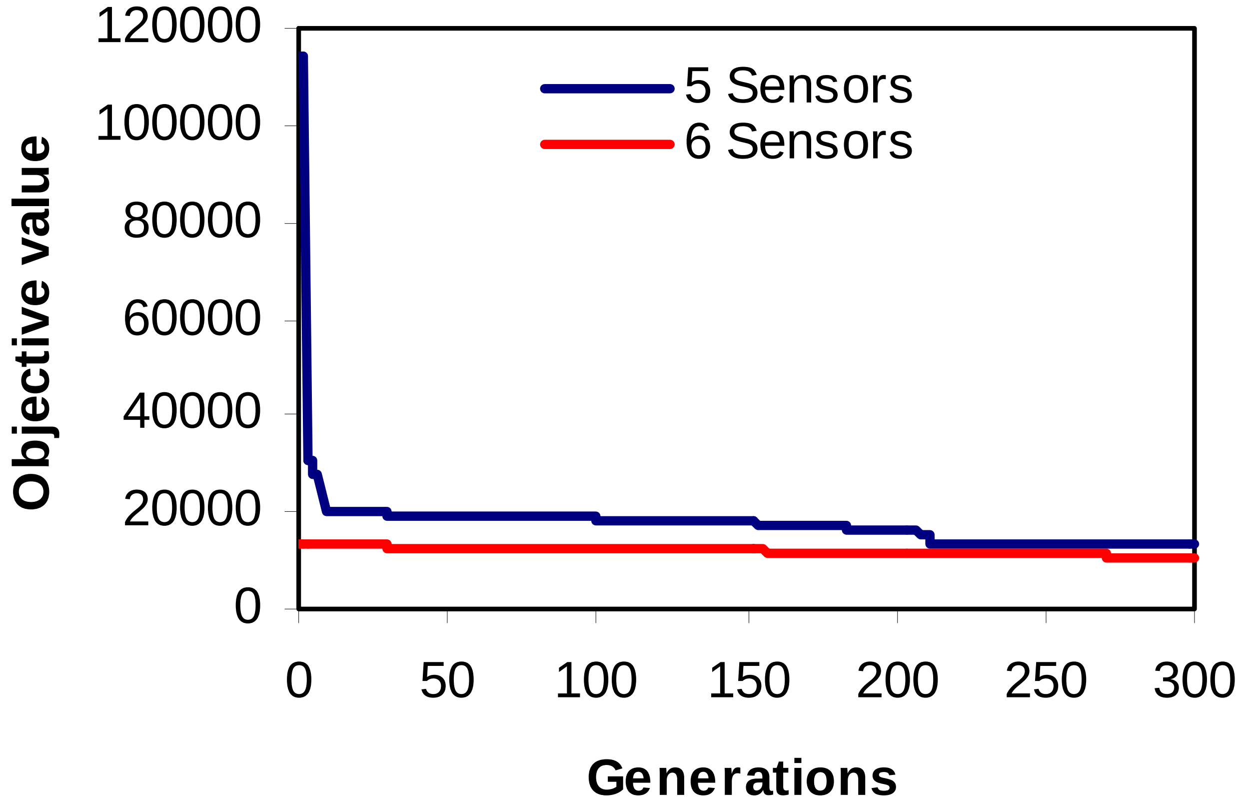
- Five sensors

Objective Function	Junction ID	Z_1 (minutes)	Z_2	Z_3 (Gal)	Z_4 (%)
The proposed	J17, J31, J81, J98, J102	409.05	200.56	3482.09	66.51
Maximizing the reliability	J11, J45, J83, J100, J117	927.64	341.37	13,117.22	81.74

Objective function value vs # of sensors



Iteration process:

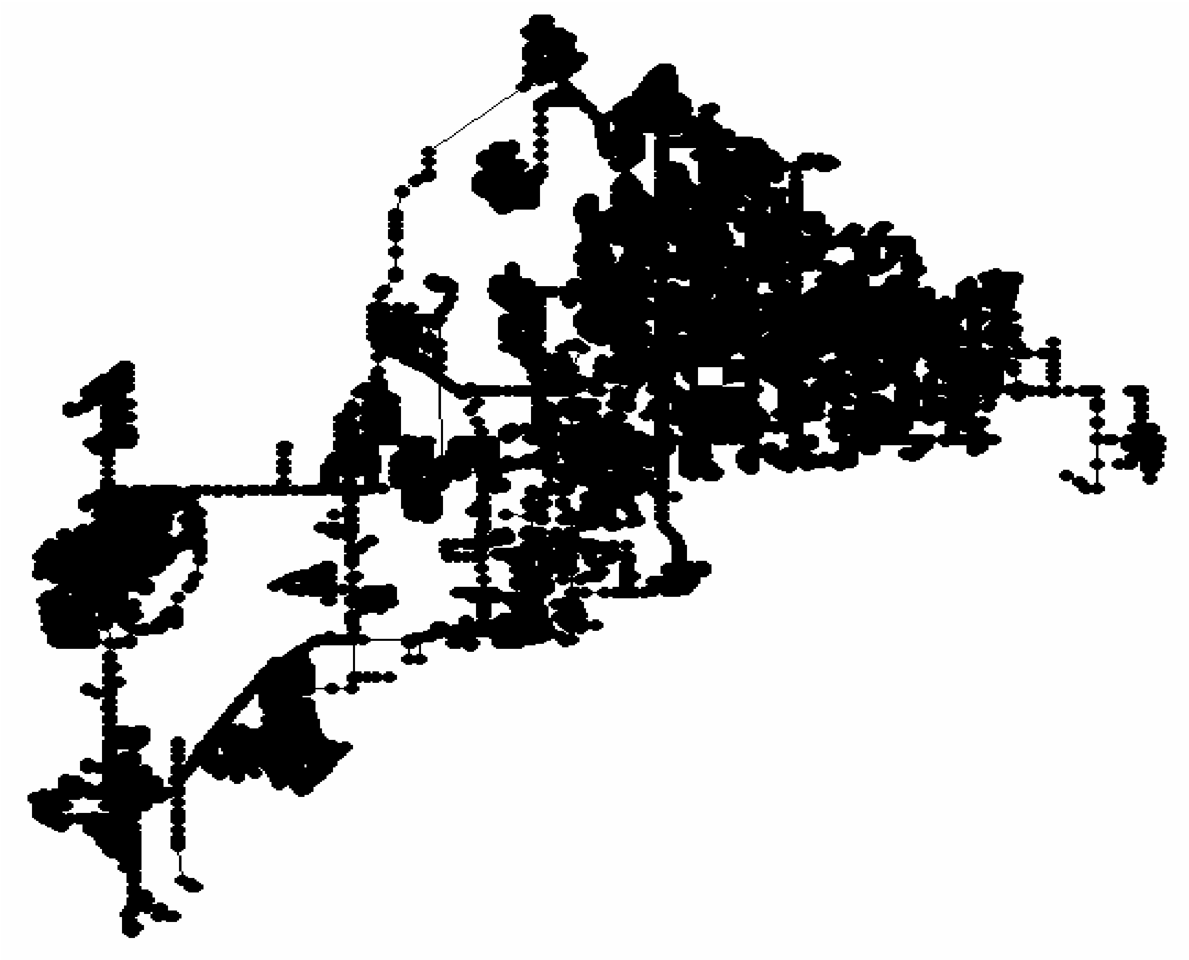


Number of sensors needed

- For 85% reliability
  - Starting iteration at M = 5
  - Five iterations used for number of sensors
  - Minimum number of sensors M = 18

Application: WDS-2

- 12,523 Junctions
- 14,822 Pipes
- 2 Reservoir
- 2 Tanks
- 4 Pumps
- 8 Valves



Scenarios used:

- 300 scenarios used in optimization which are randomly generated for the junctions with largest demands
- 1000 scenarios used in verification of solution which are generated by BWSN Utility Package

Performance:

Case		Z_1 (minutes)	Z_2	Z_3 (Gal)	Z_4 (%)
Sensors	Scenarios				
5	Optimization	163.83	1,631.86	22,361.66	22.33
	Verification	791.73	1685.57	125,468.70	21.70
20	Optimization	126.88	1,364.18	7,089.26	32.00
	Verification	645.90	925.45	41,975.72	32.10

Conclusions:

- **Single objective function**
  - Flexible algorithm
  - Good for measurement selected for objective
  - Poor for other measurements
- **Multi-objective function**
  - Trade-off coefficients directly affect the solution
  - Difficult to determine trade-off coefficients
- **Synthetic single objective function**
  - Advantages in both single and multiple objective functions

Conclusions:

- **EPANET is applied before optimization**
  - Save computational time
  - Need more computer memory
- **EPANET is used in optimization process**
  - Save computer memory
  - Need long computational time

Evaluation of performance

- **Don't Calculate Z1, Z2, Z3 for scenarios not detected**
  - Lower Z1, Z2, and Z3 which is desired
  - Lower Z4 which is not desired

- **Calculate Z1, Z2, Z3 including scenarios not detected**
  - Set the end of duration as the time of detection
  - Calculate Z2 and Z3 from injection time to the end of duration

Conclusions

- **The optimization model proposed can effectively determine minimum number of sensors and their optimal placement**
- **The objective function considers the effect of four measurement**
- **The solution methodology is efficient and convergent for solving such {0, 1} integer programming problems**
- **The water sensor network design has excellent performance**

Thank you…

For additional information or questions, you may contact:

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DATA STRUCTURE:

Water Distribution System

Junction index	Junction ID
1	Junction-0
2	Junction-1
3	Junction-2
4	Junction-2
...	...
127	Junction-126
128	Junction-127
129	Junction-128

Selected by roulette wheel

Subdomain of Junctions

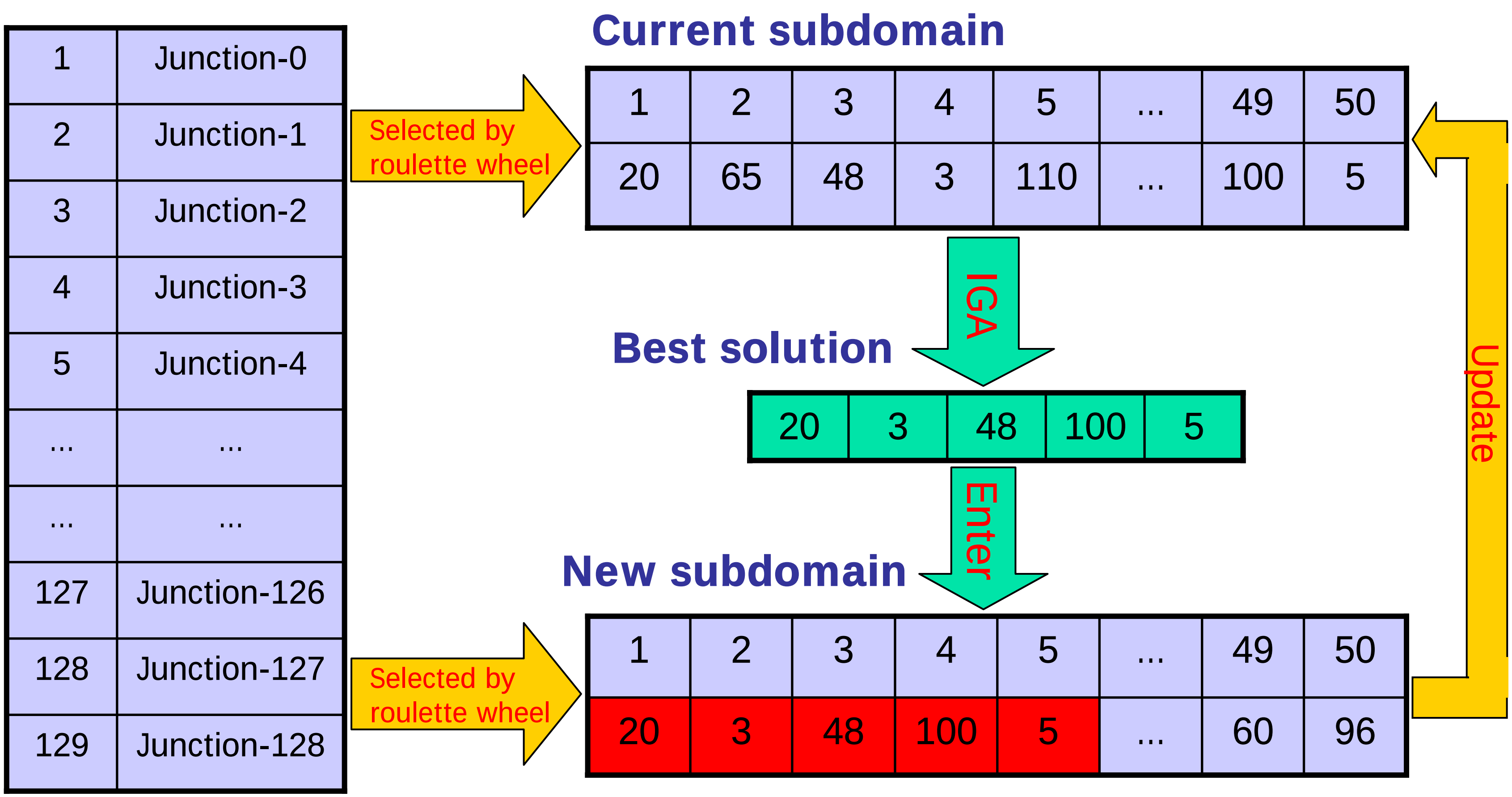
Index	1	2	...	49	50
Junction index	20	65	...	100	3

Generated by GA operators

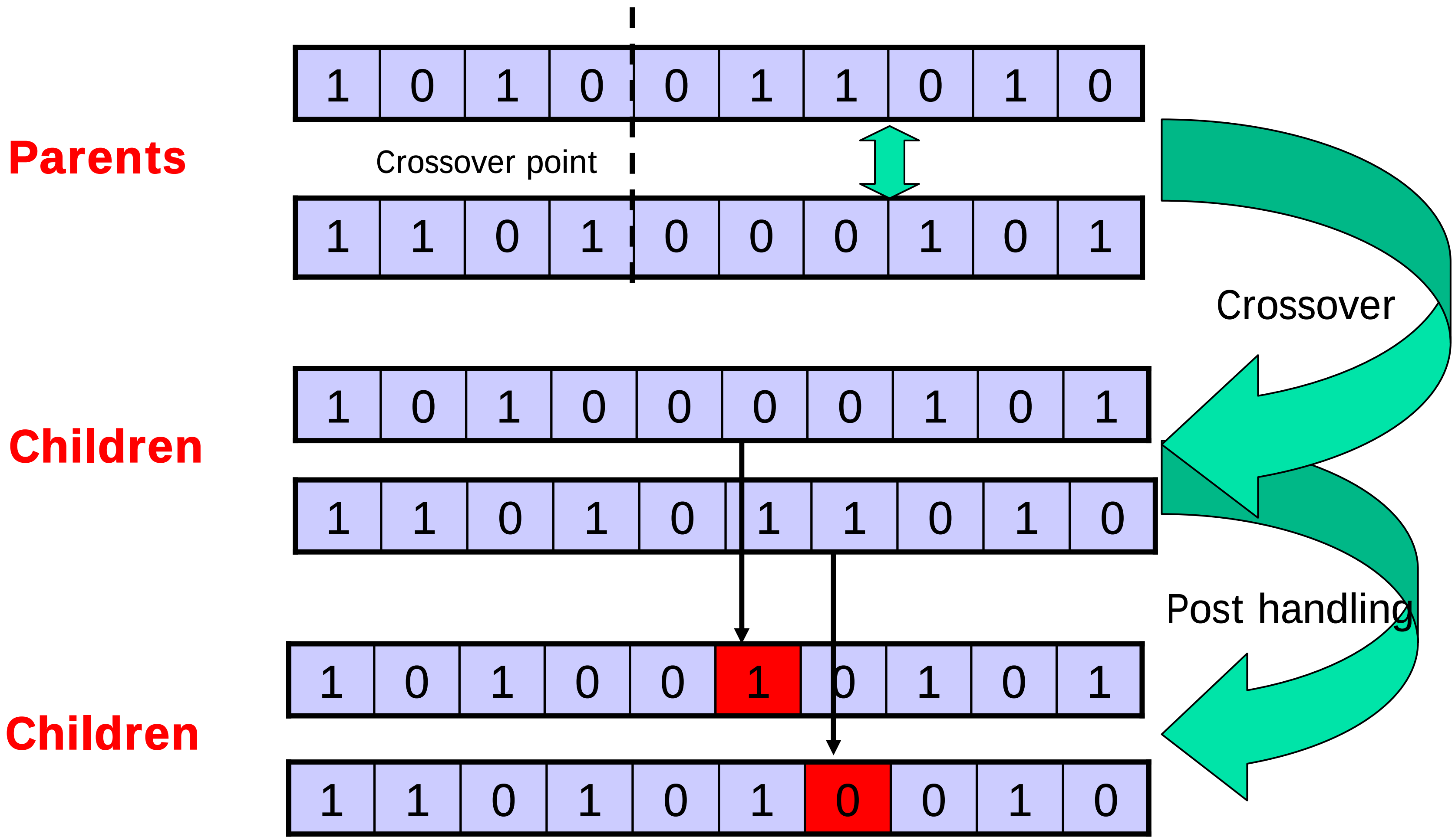
Chromosome

Bit index	1	2	...	49	50
Bit value	1	0	...	1	0

DATA STRUCTURE:



Crossover operator:



Mutation operator: